RELATING SUB-SURFACE WATER STORAGE TO STREAMFLOW IN A MOUNTAINOUS WATERSHED

by

D. Storr

ABSTRACT

Short period water balances during 74 recession periods over nine years at Marmot Creek produced a relationship between the change in sub-surface storage, streamflow and change in streamflow which by integration gave a relationship between the amount in storage and streamflow. Including the change in sub-surface storage in the August and annual water balances greatly improved their efficiency in predicting streamflow. The average annual change in storage is 49 mm. The relationships depend on the geomorphic, physiographic and climatic characteristics of the basin, so are not transposable to other areas. The method, however, is applicable to any area where short term values of precipitation, streamflow and evapotranspiration are available.

RELATION ENTRE L'EMMAGASINEMENT ET L'ÉCOULEMENT DES EAUX SOUTERRAINES DANS UN BASSIN HYDROGRAPHIQUE EN MONTAGNE

par

D. Storr

RÉSUMÉ

Les bilans hydriques pour de courtes périodes des 74 décès étudiées pendant neuf ans à Marmot Creek ont fourni une relation entre la variation de l'emmagasinement souterrain d'une part, l'écoulement et sa variation d'autre part. En intégrant, on en a tiré une relation entre la quantité emmagasinée et la quantité écoulée. En tenant compte de la variation de l'emmagasinement souterrain dans l'évaluation des bilans hydriques pour l'année et pour autoriser beaucoup d'efficacité à prévoir l'écoulement. La variation moyenne annuelle de l'emmagasinement est de 49 mm. Les relations sort fonction des caractéristiques géomorphologiques, physiographiques et climatiques du bassin et ne valent donc pas pour d'autres endroits. La méthode est cependant applicable à toute région pour laquelle on dispose de valeurs à court terme relatives aux précipitations, à l'écoulement et à l'evapo-transpiration.
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Introduction

In areas or for periods where changes in surface storage are negligible, a means of quantifying changes in sub-surface water storage is of value in the understanding of the hydrologic cycle in an area; as a check on the accuracy of measurement or estimation of the components of the water balance equation

\[ P = N + E + \Delta C_W + \Delta S_M \]  

(1)

(where P is precipitation, N is streamflow, E evapotranspiration, and \( \Delta C_W \) and \( \Delta S_M \) the changes in groundwater and soil moisture storage respectively); and in the operational problems of flood and flow forecasting. The relationship between \( \Delta C_W \) and \( \Delta S_M \) is complex and variable because of a continuous interchange of water between them, so considering the total change in sub-surface storage (\( \Delta W \)) is therefore a practical simplification, and (1) becomes

\[ \Delta W = P - N - E \]  

(2)

Preliminary studies at Marmot Creek (Storr, 1974) have indicated that the unexplained residuals in annual balances of precipitation minus streamflow and evapotranspiration (see Column 5, Table 1) correlate positively with changes in groundwater well levels (Figure 1), and amount to as much as 12% of precipitation.

To measure \( \Delta W \) directly would require detailed knowledge of soil depth and type, the porosity and permeability of the lower strata down to some impermeable layer, and a network of wells and soil moisture sites adequately distributed over the area to record the changes. From practical considerations this is not feasible in mountainous regions, so it was decided to use an indirect approach at Marmot Creek.
Site Description

Marmot Creek is a small (9.40 km²) experimental watershed at Latitude 50°57'N., Longitude 115°10'W., on the west side of the Kananaskis Valley, just east of the continental divide in Alberta. In this rugged terrain, elevations range from 1535 to 2305 metres, with an average slope of 39% causing numerous access problems. The general aspect is easterly.

The lower reaches of the basin are covered with a dense stand of lodgepole pine, then mature spruce up to 30 m tall extend to tree-line at 2135 to 2255 m. In the alpine zone, shrubs and grasses give way to talus slopes and bare rock.

The basin is relatively water-tight (Stevenson, 1967) with little or no sub-surface inflow or outflow. As reported by Jeffrey (1965), from the work of Green and Jones (1961), the relatively impermeable bedrock is exposed in the headwaters area, but almost all the remainder of the basin is covered by a deep layer of coarse and permeable surficial materials which allows rainfall to infiltrate rapidly. There is very little surface flow into the channels and the effects of rainfall on streamflow are smoothed out by its transition through sub-surface storage (Davis, 1964). The tributaries to the main creek originate as sidehill springs in the alpine zone in areas of broken sandstone and shale deposits. The soil mantle is very shallow and confined mainly to the area below tree-line. Maximum soil moisture storage is therefore a small fraction of the total sub-surface storage. There are no ponds or glaciers, so the snowpack is the only form of surface storage.

Streamflow is the most accurately measured component of the water balance at Marmot Creek. The main stem stream gauge is a concrete, 120 degrees, sharp-crested V-notch weir of 4.53m²/sec. capacity, sealed to the impermeable Rocky Mountain formation which crosses the lower end of the basin. It has been rated both volumetrically and by current meter, and conforms closely to the theoretical rating. A heat lamp keeps it ice-free in the winter.

Rainfall is measured by a network of 33 standard MSC rain gauges, and annual precipitation by a network of 11 storage and recording gauges (Storr, 1967; Ferguson and Storr, 1969). Net radiation is measured by a horizontal CSIRO sensor about 10 m above the tree-tops on a 50 m mast near the centre of the lower part of the basin.

The instrumentation network is displayed in Figure 2.
Theory

Based on the accepted theory (Linsley, Kohler and Faulhus, 1949) that streamflow during recession periods represents withdrawal of water from storage, initial attempts (Stevenson and Davis, 1967) were made to solve the problem of quantifying the change in storage at Marmot Creek by correlating streamflow during recession periods with the change in height of the water level in many of the 17 wells in the basin, both singly and in combination. These attempts failed to find a consistent relationship, possibly because of inadequate sampling of groundwater in this geologically complex area, but also because the variable permeability and porosity of the surficial materials resulted in highly variable responses and lag times of the various wells, destroying the comparability of the data.

In this study a different approach was taken. Recession discharge does not represent the total loss by the sub-surface reservoir; it is also losing water by evapotranspiration and gaining from precipitation, as shown in (2). ∆W was therefore calculated for the recession periods used in the initial approach and plotted against the change in height of the water table in the wells. Much of the former variability was removed, thus strengthening confidence in the validity of the new method, but the weakness of the well data remained. Maintenance of most of the wells was discontinued in 1971.

Therefore another approach to the problem of quantifying sub-surface storage was developed. At any time when there is no surface flow into the stream, the rate of streamflow is an index of the amount in the groundwater reservoir. The higher the level in the reservoir, the greater the hydraulic head, and the greater the discharge from groundwater to the stream, at all points on the channel. The streamflow at such times is therefore the integration of groundwater discharge into the channel over all the upstream area, and is a function of the amount in the reservoir. The change in discharge over a period is therefore an index of the change in groundwater storage. Because groundwater storage is much greater than soil moisture storage in this basin and they tend to increase or decrease together, streamflow is also an index of the total sub-surface storage and the change in streamflow (∆N) is an index of the change in storage.

After some investigation it became apparent that the relationship between the change in streamflow and change in storage was not simple, and that the addition of another variable N (the mean of the daily streamflows N₁, N₂) at the beginning and end of the period was required. If the reservoir is considered as roughly shaped like an inverted and skewed bowl, when it is nearly full (and N is therefore high), a unit change in N represents a much smaller change in storage than a unit change in N when the reservoir is nearly empty (and N is low).
The problem then became one of isolating periods so that the relationships between $\Delta N$, $\bar{N}$ and $\Delta W$ could be determined.

Procedure

To reduce changes for error in calculating $\Delta W$ from (2), 74 recession periods were selected from nine years of record. Periods containing snowfall or melting of the snowpack were excluded, as were any beginning or ending in rain. A basin average of $P$ was calculated from the data from the 33 rain gauges, using Thiessen polygon weighting factors. For a number of periods, as a check, isohyetal averages were also calculated, but with such a dense network the difference was less than 3% and did not justify the additional work and the semi-subjective factor introduced. In the periods considered, $P$ was usually the smallest of the three components of $\Delta E$, so errors in its estimation have a minimal effect. Rainless periods were ideal, completely eliminating this source of error.

An estimate of the basin average of $E$ for each period was obtained by the energy budget method described by Storrs (1974). In brief, this involved calculation of the basin average of effective positive net radiation (Storrs, 1972), subtracting 2 - 3% for photosynthesis in the growing season (Demmead, 1964), and 14 - 17% for the energy flux into the ground and forest (Rauner, 1958; Baurgartner, 1956), and the separation of the remaining fluxes of latent and sensible heat by an estimated annual curve for $L(1+0.1B)$, where $L$ is the latent heat of evaporation and $B$ is Bowen's ratio. $E$ was always the largest component of $\Delta W$, so errors in its estimation have a direct effect on $\Delta W$. Period length varied from 5 to 15 days, helping to smooth out errors in estimating $E$.

Values of $N$, $\Delta N$ and $\bar{N}$ were obtained from the main weir hydrometric data. In recession periods, $\Delta N$ was considered negative to correspond with $\Delta W$, and varied from 1 to 5 cfs (0.02412 to 0.1416 m$^3$/s). This size reduced errors due to the limitations of accuracy of the stage discharge rating curve.

Results and Discussion

The computer-determined relationship between $\Delta W$, $\Delta N$ and $\bar{N}$ (in metric units):

$$\Delta W (\text{mm}) = 2154e^{-3.11\bar{N}} \Delta N$$

is shown in Figure 3. The relatively small amount of scatter from the best-fit curve suggests that any errors are either very small, or are self-cancelling over the length of time in each period. It is suspected that most of the scatter is due to variations from the mean Bowen ratio...
used in the evapotranspiration estimates. The actual Bowen ratio will fluctuate around the mean depending on the availability of soil moisture, so some of the 74 evapotranspiration estimates will be too high and some too low.

The form of the relationship suggests an exponential hydrograph recession of the form:

$$N_t = N_0 e^{-Kt}$$  \hspace{1cm} (4)

where $t$ is time units after the reference discharge $N_0$, and the recession coefficient $K$ is a function of $N$ as proposed by Apollon et al (1960). Examination of hydrographs for Marmot Creek confirms a linear recession relationship on semi-log paper with the slope of the recession decreasing with flow, but this relationship is outside the purpose of this paper and has not been pursued further.

The relationship between $\Delta W$, $\Delta N$ and $N$ depends on the geomorphic and physiographic characteristics of the basin, such as drainage area, size and shape, mean slope, condition of the channels, vegetative cover, the permeability and porosity of the sub-surface, and on the amount of evapotranspiration, so is not transposable to other basins with different characteristics. The method however can be applied to any drainage area for which short term values of discharge, evapotranspiration and precipitation are available.

Assuming that $W = 0$ at $N = 0$, and taking small enough $\Delta W$ and $\Delta N$, integration of (3) gives:

$$W(\text{mm}) = 265.6 \left(1 - e^{-3.11N}\right)$$  \hspace{1cm} (5)

For very small $\Delta N$, $N \to N_0$, so

$$W(\text{mm}) = 265.6 \left(1 - e^{-3.11N}\right)$$  \hspace{1cm} (6)

which is illustrated in Figure 4.

This relationship indicates a maximum reservoir capacity of 265 mm which is reached at a flow of approximately 0.7 m$^3$/s, and has practical applications in flow and flood forecasting. As an example, at $N = 0.4$ m$^3$/s, the storage level is 255 mm, another 10 mm of rainfall or snowmelt would saturate the basin, and any excess would become direct surface flow into the channels. However at $N = 0.2$ m$^3$/s, $W = 213$ mm and 52 mm of storage are available before there is any runoff excess. At Marmot Creek, capacity by this criterion is usually reached in June from the combination of rapid snowmelt and heavy rainfall. For the rest of the year, storage is below capacity, and the high infiltration rate prevents direct flow into the channels.
The use and value of calculating $\Delta W$ are illustrated in Tables 1 and 2, the water budgets for 10 water-years and 10 Augments. These periods were chosen because the snow storage problem is eliminated. It can be seen from both tables that the net errors are almost equally divided between positive and negative values, suggesting that there is no obvious bias in any of the budget components. In the annual budget without the inclusion of $\Delta W$, the maximum unexplained residual is 11.6% of precipitation and the average is 6%. Including $\Delta W$ reduces the maximum error to 5% and the average to 1.7%. From a statistical viewpoint, the efficiency $R^2$ (as defined by Nash and Sutcliffe, 1970) of the annual budget in predicting discharge is -0.08 without $\Delta W$, but +0.91 when $\Delta W$ is included. In the August balances, $R^2$ without including $\Delta W$ is a very inadequate -0.64 but becomes more respectable at +0.73 when $\Delta W$ is included. Again it is suspected that the relatively large errors in the August balances are caused by deviations from the assumed mean Bowen ratio in the evapotranspiration estimate. Further work is underway to improve this factor. It is noted in Table 1 that the average annual change in sub-surface storage of 49 mm indicates that a year is not a long enough period for this component of the water balance to be neglected.

The calculation of $\Delta W$ has also been of value in a water balance evaluation of snow survey data at Marmot Creek (Storr and Golding, 1973).

The next phase of hydrometeorological investigation at Marmot Creek will be to attempt the development of similar water balances for each of the sub-basins. When some of the forest is removed from one sub-basin in the manipulative phase of the project, the change in albedo will change the amount of evapotranspiration and hence the $\Delta W$, $\Delta N$, $\tilde{N}$ relationship and the water balance of that sub-basin. The calculation of the amount of this change is complex but will be possible when the change in albedo has been measured.

Conclusions

The relationships between $W$, $\Delta W$, $\tilde{N}$, and $\Delta N$ developed for Marmot Creek should not be assumed applicable to other streams with different physiographic characteristics and in a different climate. Although $\Delta N$ by itself will indicate whether the amount of water in storage has increased or decreased in the period, to quantify the change requires estimates of precipitation and evapotranspiration for relatively short periods. For many areas this presents many problems. Further
study on the reliability of using pan data or empirical formulae for evapotranspiration and index stations for precipitation is necessary.

Acknowledgements

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APPROVED

J. R. H. Noble
Assistant Deputy Minister
Atmospheric Environment Service.
References


### Table 1

ANNUAL WATER BALANCES AT MARMOT CREEK (millimeters, except $N_1$, $N_2$ in $m^3/s$)

<table>
<thead>
<tr>
<th>PERIOD</th>
<th>P</th>
<th>N</th>
<th>E</th>
<th>RESIDUAL</th>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$\Delta N$</th>
<th>NET ERROR</th>
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<td>15/10-63-30/09/64</td>
<td>1039</td>
<td>488</td>
<td>438</td>
<td>+113</td>
<td>.0408</td>
<td>.1387</td>
<td>+102</td>
<td>+11</td>
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<td>30/09/64-02/10/65</td>
<td>1027</td>
<td>520</td>
<td>493</td>
<td>+ 19</td>
<td>.1387</td>
<td>.1900</td>
<td>+ 29</td>
<td>- 10</td>
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<td>02/10/65-03/10/66</td>
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<td>495</td>
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<td>.2900</td>
<td>.0476</td>
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</tr>
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<td>.0377</td>
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<td>-43</td>
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<td>05/10/67-10/09/68</td>
<td>855</td>
<td>353</td>
<td>407</td>
<td>+ 95</td>
<td>.0377</td>
<td>.1104</td>
<td>+ 86</td>
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<td>- 55</td>
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<td>.0459</td>
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<td>.0459</td>
<td>.0425</td>
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<td>-10</td>
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<tr>
<td>08/09/70-07/09/71</td>
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<td>445</td>
<td>453</td>
<td>+ 11</td>
<td>.0425</td>
<td>.0442</td>
<td>+ 5</td>
<td>+ 8</td>
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<tr>
<td>07/09/71-05/09/72</td>
<td>972</td>
<td>487</td>
<td>435</td>
<td>+ 50</td>
<td>.0442</td>
<td>.0683</td>
<td>+ 33</td>
<td>+17</td>
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<tr>
<td>05/09/72-04/09/73</td>
<td>852</td>
<td>414</td>
<td>415</td>
<td>+ 23</td>
<td>.0683</td>
<td>.0918</td>
<td>+ 26</td>
<td>- 3</td>
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<td>Absolute Total</td>
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<td>4490</td>
<td>4376</td>
<td>536</td>
<td>490</td>
<td>150</td>
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<td>53.6</td>
<td>49.0</td>
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<td>Algebraic Total</td>
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<td>64</td>
<td>18</td>
<td></td>
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### Table 2

AUGUST WATER BALANCES AT MARMOT CREEK (millimeters, except $N_1$, $N_2$ in $m^3/s$)

<table>
<thead>
<tr>
<th>YEAR</th>
<th>P</th>
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<th>E</th>
<th>RESIDUAL</th>
<th>$N_1$</th>
<th>$N_2$</th>
<th>$\Delta N$</th>
<th>NET ERROR</th>
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<td>1964</td>
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<td>.1189</td>
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<td>1965</td>
<td>122.4</td>
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<td>.1742</td>
<td>.1320</td>
<td>-26</td>
<td>-10.1</td>
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<tr>
<td>1966</td>
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<td>79.7</td>
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<td>.0689</td>
<td>-38</td>
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<td>1967</td>
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<td>25.3</td>
<td>100.6</td>
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<td>.1320</td>
<td>.0510</td>
<td>-83</td>
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<td>.1133</td>
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<td>25.7</td>
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<td>.0552</td>
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<td>Algebraic Total</td>
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* Partial month due to missing radiation data Aug. 8 - 12.
Figure 1. Relationship between annual water balance residuals (R) and mean change in groundwater well levels (ΔH).
\[ \Delta W = 2154.4 \Delta N (e^{-0.1\bar{N}}) \]

\[ r^2 = 0.939 \]

Figure 3. Relationship between \(\Delta W\), \(\Delta N\), and \(\bar{N}\) at Marmot Creek.
Figure 4. Relationship between subsurface storage (W) & streamflow (N) at Marmot Creek.

\[ W = 265.65 (1 - e^{-B \cdot I N}) \]