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Estimating areal snowmelt infiltration into frozen soils

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Abstract:

An algorithm for estimating areal snowmelt infiltration into frozen soils is developed. Frozen soils are grouped into classes according to surface entry condition as: (a) *Restricted*—water entry is impeded by surface conditions, (b) *Limited*–capillary flow predominates and water entry is influenced primarily by soil physical properties, and (c) *Unlimited*—gravity flow predominates and most of the meltwater infiltrates. For *Limited* soils cumulative infiltration over time is estimated by a parametric equation from surface saturation, initial soil moisture content (water + ice), initial soil temperature and infiltration opportunity time. Total infiltration into *Unlimited* and *Limited* soils is constrained by the available water storage capacity. This constraint is also used to determine when *Limited* soils have thawed.

The minimum spatial scale of the infiltration model is established for *Limited* soils by the variabilities in surface saturation, snow water equivalent, soil infiltrability, soil moisture (water + ice) and depth of soil freezing. Since snowmelt infiltration is influenced by other processes and factors that affect snow ablation, it is assumed that the infiltrability spatial scale should be consistent with the scales used to describe these variables. For open, northern, cold regions the following order in spatial scales is hypothesized: frozen ground $\geq$ snowmelt $\geq$ snow water equivalent $\geq$ frozen soil infiltrability $\geq$ soil moisture (water + ice) and snow water.

For mesoscale application of the infiltration model it is recommended that the infiltrability scale be taken equal to the scale used to describe the areal extent and distribution of the water equivalent of the snowcover that covers frozen ground. Scaling the infiltrability of frozen soils in this manner allows one to exploit established landscape-stratification methodology used to derive snow accumulation means and distribution.

Scaling of soil infiltrability at small scales (microscale) is complicated and requires information on the association(s) between the spatial distributions of soil moisture (water + ice) and snow water.

A flow chart of the algorithm is presented. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS frozen soils; simulation; scaling; soil infiltrability; snow ablation

INTRODUCTION

In northern regions, melting of the seasonal snowcover is one of the most important events of the water year. Water from melting snow supplies reservoirs, lakes and rivers and recharges soil moisture and groundwater storage. Partitioning of the snow water resource to runoff and soil water largely depends on the infiltrability of the underlying frozen ground at the time of snowmelt.

Infiltration of meltwater into frozen soil is a complicated process as it involves coupled heat and mass flow with phase changes. The thermal and hydrophysical properties of the soil, the soil temperature and moisture regimes, and the quantity and the rate of release of meltwater from the snowcover affect the process. Numerous investigations of snowmelt infiltration into frozen soil are reported in the literature. These include (a) field studies of the phenomenon in Alaska (Kane, 1980; Kane and Stein, 1983), in the Canadian Prairies (Granger *et al*., 1984; Gray *et al*., 1986b), and in the central Yukon territory of Canada (Burn, 1990),

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(b) theoretical and numerical analyses of coupled heat and mass flow (Motovilov, 1979; Lundin, 1990; Grant, 1992; Engelmark and Svensson, 1993; Tao and Gray, 1994; Zhao *et al*., 1997) and (c) the development of long-term forecasts of seasonal hydrological processes (Popov, 1972; Kuchment *et al*., 1986; Flerchinger and Saxton, 1989; Illangasekare and Walter, 1990). Despite the advancements made in understanding the process, the methodologies and procedures to be followed for proper implementation of this knowledge into operational hydrological simulations have not been fully described.

During the past two and a half decades researchers in the Division of Hydrology, University of Saskatchewan and the National Hydrology Research Institute at Saskatoon have conducted extensive studies of snow accumulation and wind transport of snow, ablation of seasonal snowcovers, and coupled heat and mass transfer in snow and underlying frozen ground. These investigations included field measurements at sites in prairie, boreal forest, alpine and arctic environments and associated theoretical work of various processes. The findings of many of these studies can be found in the publications by Male (1980), Gray and Male (1981), Pomeroy and Gray (1995) and Pomeroy *et al*. (1998) and cited literature. This paper uses the experience and knowledge gained in this work, together with the findings of recent research as a framework, to develop an algorithm for estimating areal snowmelt infiltration into frozen soils.

INFILTRATION MODEL

Infiltration involves water transmission and storage; therefore the process is influenced by all factors that affect both the entry of water at the surface and the downward and lateral movement of the wetting front within the soil profile. Gray *et al*. (1986a), following the suggestions of Popov (1972), proposed that frozen soils can be separated into three general *'groups'* in terms of their infiltrability for snowmelt based on their surface entry condition (see Figure 1).

1. *Unlimited* (predominately gravity flow): soils are capable of infiltrating most or all available meltwater. This group includes:

   - Dry, severely cracked colloidal soils—usually fine-textured mineral soils under vegetation during the summer months.

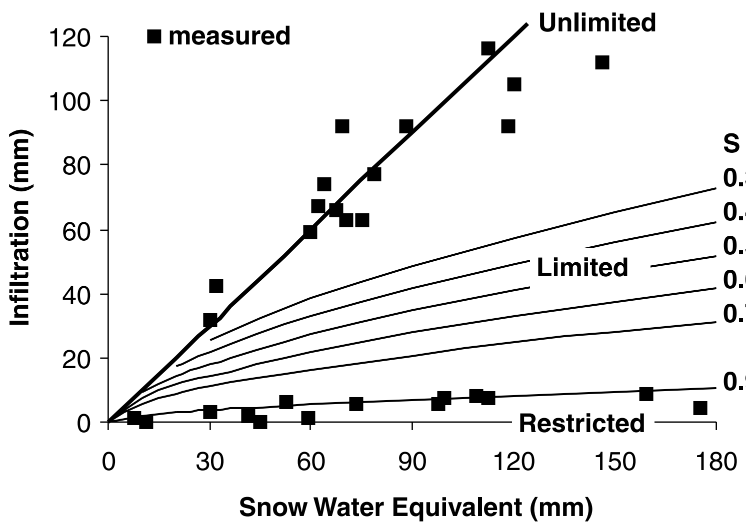

Figure 1. Infiltration versus snow water equivalent for *Unlimited, Limited* and *Restricted* frozen soils

- Dry, coarse-textured and gravelly soils—deep, relatively-uniform profiles and sand, sandy loam or other permeable material underlying well-drained peat and organic deposits (e.g. forest soils).
- Sloping, shallow, highly permeable deposits underlain by a relatively impermeable subsurface material (e.g. forest litter and moss on bedrock, tundra peat on permafrost).

2. *Restricted*: soils whose infiltrability is restricted by an impervious surface. This group includes:

- Mineral and other soils frozen in a wet condition (e.g. >70–80% pore saturation).
- Soils in which an impeding layer develops at/near the ground surface. For example, a basal ice lens that forms at the soil surface due to rain that occurs near freeze-up or the freezing of percolating or infiltrating meltwater.

3. *Limited* (predominately capillary flow): soil infiltrability is governed primarily by the soil moisture content (water + ice) and soil temperature at the start of snow ablation and the infiltration opportunity time.

Formulations for limited soils

Zhao and Gray (1999) reported a parametric equation describing cumulative infiltration into frozen, unsaturated soils of *Limited* infiltrability, INF (mm) as:

$$\mathrm{INF} = CS_0^{2\cdot92}(1 - S_\mathrm{I})^{1\cdot64}\left(\frac{273\cdot15 - T_I}{273\cdot15}\right)^{-0\cdot45} t_0^{0\cdot44} \tag{1}$$

where C = coefficient, S_0 = surface saturation—moisture content at the soil surface (mm^3/mm^3), S_I = average soil saturation (water + ice) of 0–40 cm soil layer at the start of infiltration (mm^3/mm^3). $S_\mathrm{I} = \theta_\mathrm{I}/\phi$, where θ_I = average volumetric soil moisture (water + ice) at start of infiltration (mm^3/mm^3) and ϕ = soil porosity (mm^3/mm^3). T_I = average temperature of 0–40 cm soil layer at the start of infiltration (K), and t_0 = infiltration opportunity time (h).

The classification of infiltrability of frozen soils described above considers the rate that a frozen soil infiltrates meltwater with the minimum rate set by the *Restricted* condition (dINF/d$t \cong 0$) and the maximum by the *Unlimited* condition (dINF/dt = rate of delivery of meltwater to the ground surface). For soils with *Limited* infiltrability, a change in infiltration by Equation (1) in an incremental increase in infiltration opportunity time estimates dINF/dt.

A first generation model that employed the above infiltrability grouping was used to calculate runoff from snowmelt from a small watershed in western Saskatchewan (Gray *et al.*, 1985). Seasonal (total) infiltration amounts calculated as the difference between measured snow water equivalent and streamflow were in good agreement with model estimates. The study used Equation (7) to estimate seasonal infiltration into *Limited* soils.

Storages

The maximum amount of snow water a frozen soil of *Unlimited* class can infiltrate is constrained by its available soil water storage, which on flat land is equal to the air-filled porosity above an impeding or relatively impermeable layer. The water storage potential, W_sP (mm) is calculated from the porosity, ϕ (mm^3/mm^3), the average initial saturation (prior to infiltration), S_I (mm^3/mm^3) and the depth (as measured from the ground surface), z_p (mm) of the highly permeable surface layer (e.g. thickness of organic layer and depth of surface-connected cracks) as:

$$W_\mathrm{sP} = \phi(1 - S_\mathrm{I})z_\mathrm{p} \tag{2}$$

The product, $\phi(1 - S_\mathrm{I})$, in Equation (2) is the air-filled porosity.

A frozen soil of *Limited* class in which capillary flow predominates does not normally become saturated throughout its wetted depth by infiltrating meltwater because ice blocks the interconnectivity of soil pores and the wetting front advances ahead of an overlying layer of higher moisture (see Figure 2a and b). Gray *et al.* (1986a) found in frozen mineral agricultural soils of the Canadian Prairies that the upper limit of saturation, L (mm^3/mm^3) of the soil layer wetted by infiltrating meltwater during snow ablation is related to the average initial saturation of the layer as:

$$L = 0{\cdot}6 + 0{\cdot}4S_\mathrm{I} \tag{3}$$

Combining Equations (2) and (3) gives:

$$W_\mathrm{sP} = 0{\cdot}6\phi(1 - S_\mathrm{I})z_\mathrm{w} \tag{4}$$

in which z_w is the depth of wetting. Equation (4) suggests that the average storage potential of a frozen fine-textured mineral soil is of the order of 60% of its air-filled pore space at the start of infiltration. Replacing z_w in Equation (3) by the depth of frost, z_f, provides a measure of the potential available storage of snowmelt water of a *Limited* soil in the frozen state.

Application of formulations

When applying the infiltration model the cumulative infiltration should be tested against the limiting value for W_sP at the end of each time step. For soils with *Limited* infiltrability this comparison is used to determine whether a soil had thawed during infiltration. Thawing enhances infiltration because of the changes to the thermo- and hydrophysical properties of a soil and to the state and gradients that govern coupled heat and mass transfers that accompany a phase change. Figure 3 provides field measurements to illustrate this point. These measurements were made during snow ablation in the spring, 1999 in a sandy loam soil at a site in the boreal forest near Waskesiu, Saskatchewan. Figure 3a shows the soil temperature profile at

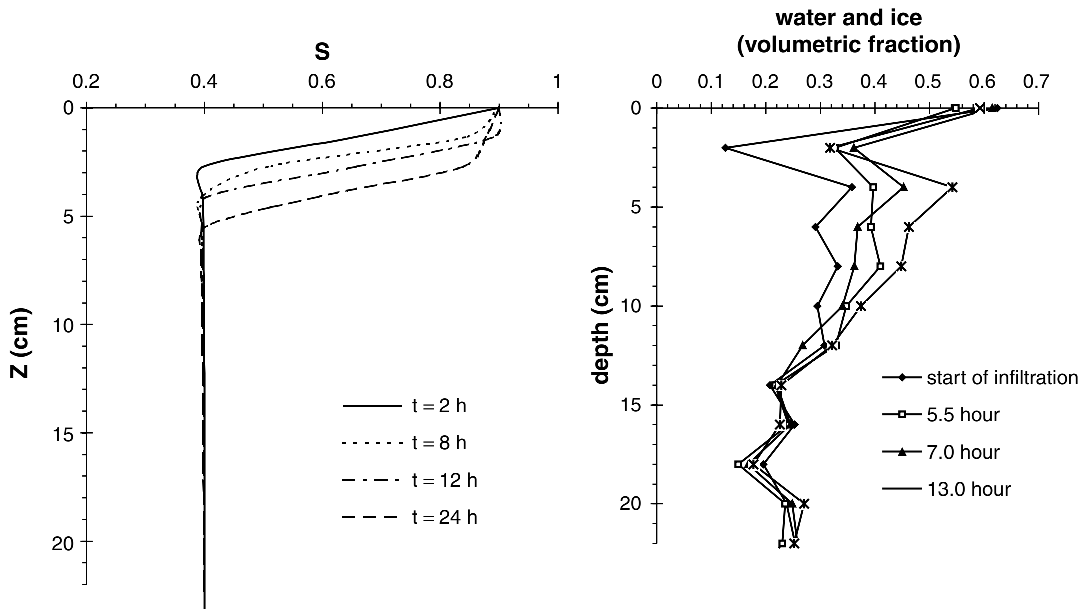

Figure 2. Profiles of the variation in soil saturation, S, with depth at various times during meltwater infiltration: (a) simulated profiles into a homogeneous frozen soil initially with uniform moisture (water + ice) distributions, under continuous infiltration and a constant head at the surface: $S_\mathrm{I} = 0{\cdot}4$, $S_0 = 0{\cdot}75$ and $T_\mathrm{I} = 269$ K and (b) measured soil moisture profiles in a sandy loam during interrupted snowmelt infiltration at a site in the boreal forest near Waskesiu in the spring, 1999

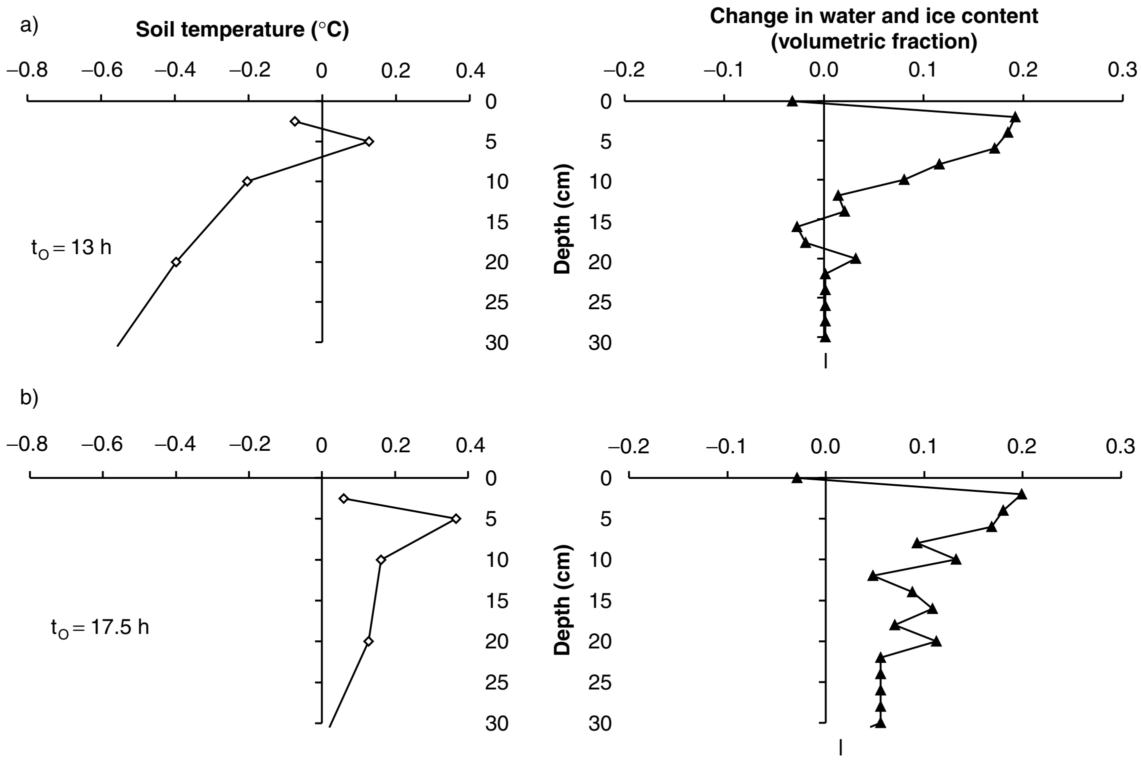

Figure 3. Profiles of soil temperature and change in soil moisture (water + ice) monitored during snow ablation in a sandy loam soil at a site in the boreal forest near Waskesiu in the spring, 1999. Part (a) shows the soil temperature profile at 1800 h March 26/99 and the change in soil moisture (water + ice) between 1200 h March 25/99 and 1800 h March 26/99. Part (b) shows corresponding measurements at the site at 0830 h the next day, March 27/99

1800 h March 26/99 and the change in soil moisture (water + ice) monitored at the site between 1200 h March 25/99 and 1800 h March 26/99. The measurements suggest that most of the soil layer, 0–30 cm, was frozen and that infiltrating meltwater had penetrated the frozen soil to a depth of about 14·5 cm. The moisture change represents about 15·5 mm of infiltration in a total infiltration opportunity time of about 13 h. Figure 3b shows corresponding measurements at the site at 0830 h the next day, March 27/99. Overnight the soil profile warmed—the temperature measurements suggest it thawed and/or became isothermal to 0 °C (within measurement error—thermistor precision = ±0·15 °C)—and waters percolated below 30 cm. The measured change in moisture between 1200 h March 25/99 and 0830 h March 27/99 was 28·8 mm and the total infiltration opportunity time was estimated as 17–18 h. That is, about 13·3 mm of water infiltrated in 4–5 h. Much of the melting and some of the meltwater were the result of a light (~1·3 mm) rain shower that persisted to 2200 h, March 26 and warm air temperatures.

Komarov and Makarova (1973) suggest that a soil frozen to a depth less than 150 mm behaves the same as an unfrozen soil in respect of infiltration and once it is frozen to 600 mm, freezing to greater depth has no further effect on the process. The assignment of a specific frozen depth to decide whether a soil remains frozen throughout ablation is arbitrary and excludes (directly) many of the factors that affect the transfers of heat and mass during infiltration. This paper proposes that once infiltrating meltwater has satisfied the water storage potential of the frozen depth, a soil behaves as an unfrozen soil in respect to infiltration.

Soil texture

It is appropriate to comment briefly on the influence of soil texture on the infiltrability of *Limited* frozen soils because soil morphological and textural properties are commonly used as spatial indices of the water transmission and storage properties of unfrozen ground. For example, in land classification schemes for irrigation, drainage or runoff. Zhao *et al.* (submitted) report the results of field and model studies on the association between soil texture and snowmelt infiltration into frozen soils. Their findings show that the infiltrability of frozen soils is relatively unaffected by soil texture. On the basis of these results, this paper assumes that the effects of spatial variations in soil texture on scaling the infiltrability of frozen soils can be ignored (except as texture may influence initial soil saturation and soil porosity). The poor correlation between frozen soil infiltrability and soil texture suggests that the process is primarily affected by state variables, i.e. the soil moisture (water + ice) content and soil temperature and their gradients, rather than on the physical and thermal properties of the soil matrix.

DEVELOPMENT OF ALGORITHM FOR AREAL SNOWMELT INFILTRATION INTO FROZEN SOILS

Spatial considerations

Snowmelt infiltration into frozen ground is affected by the thermal and hydrophysical properties of the soil, the soil temperature and moisture regimes, and the rate and quantity of meltwater released from the snowcover. Since the conditions for the phenomenon are frozen ground, snowcover and snowmelt, the infiltrability scale should be consistent with the scales used to describe these processes. An order in scales among primary parameters is hypothesized. For example, in open, relatively flat cold northern regions the minimum scale required to define the areal extent and variability of frozen ground is the maximum scale for snowmelt; the minimum scale required to define the spatial variability of snowmelt is the maximum scale for snow water equivalent; and the minimum scale required to define the spatial variability of snow water equivalent is the maximum scale for frozen soil infiltrability. Further partitioning of an area depends on the variabilities of the primary factors affecting frozen soil infiltration and snowcover depletion. This order is not intended to negate the possibility that all scales could be taken equal nor that the prescribed order applies in all cold climate environments.

The smaller spatial scales for frozen soil infiltration are likely to be set by the spatial variability of the primary factors affecting the process within areal units whose soils have been designated as *Limited* infiltrability. These variables are (Equations (1) and (4)): the coefficient, C (see discussion below) and S_0, S_I, T_I, t_0, ϕ and z_f. Obviously, the practicality and feasibility of fine-tuning the calculations of all variables in order to scale frozen soil infiltration is put to question, especially where the objective is to *'upscale'* the infiltration process. Elimination of those variables that can be safely ignored from the procedure can be achieved by testing the sensitivity of the process to changes in a variable. Further refinement of scale will depend on the sensitivity of the phenomenon under investigation and/or simulation to variations in infiltrability.

SENSITIVITY OF INFILTRATION ESTIMATES BY PARAMETRIC EXPRESSION (EQUATION (1)) TO VARIATIONS IN ITS COMPONENTS

Coefficient C

The coefficient C (Equation (1)) characterizes the effects on infiltration of differences between model and natural systems. For example, the expression (Equation (1)) assumes quasi-steady flow (Figure 4) when surface saturation is constant, a soil is homogeneous, and the vertical distributions of soil moisture and soil temperature at the start of infiltration are uniform and constant. Figure 5 compares calculated (Equation (1), $S_0 = 1{\cdot}0$) and measured seasonal snowmelt infiltration into fine-textured (sandy loam, loam, silty clay and clay) frozen soils monitored at various sites in the Brown and Dark Brown soil zones of Saskatchewan (Figure 5a) and for sandy soils at a site in the boreal forest near Waskesui. For the prairie soils $C = 2{\cdot}10$ with a standard deviation of the differences between calculated and measured values,

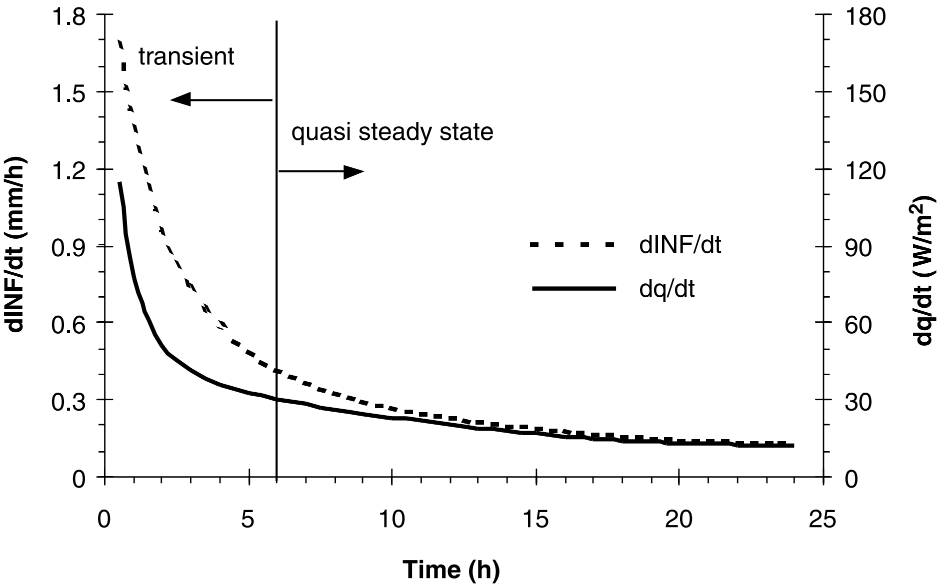

Figure 4. Variations in infiltration rate, dINF/dt, and surface heat flux rate, dq/dt, with time during snowmelt infiltration into a frozen silty clay soil

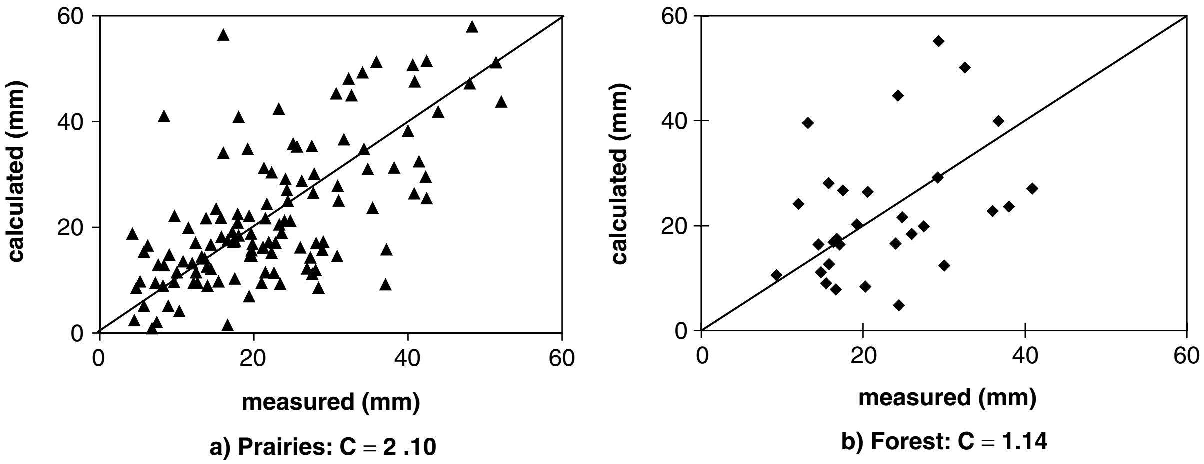

Figure 5. Seasonal infiltration calculated by the parametric expression (Equation (1)) with $S_0 = 1$ for the prairie environments (a) and forest (b)

$s_d = \pm 10{\cdot}5$ mm and for the forest soils $C = 1{\cdot}14$ and $s_d = \pm 12{\cdot}1$ mm. C can be adjusted for a decrease in surface saturation as: $C_{S_0} = C_1/S_0^{2{\cdot}92}$, where C_{S_0} = coefficient for surface saturation, S_0, and C_1 = coefficient for $S_0 = 1{\cdot}0$.

The variation in C with constant S_0 in different environments confines the use of Equation (1) for estimating frozen soil infiltration when the cause(s) for the variation are unknown. Therefore, a study of the effects on infiltration of the vertical distribution of soil moisture (water + ice) in the soil profile at the start of infiltration was undertaken.

EFFECTS OF THE VERTICAL DISTRIBUTION SOIL MOISTURE AT START OF INFILTRATION

Normally, information on the soil moisture regime in the fall of a year would be used to estimate S_I, since most soil models do not calculate changes in soil moisture during winter. Vertical profiles of soil moisture may exhibit diverse patterns due to variations in climate history and land use. On cropped land, the soil moisture content usually increases with increasing depth because of withdrawal of water for evapotranspiration during the growing season. Conversely, rainfall near the start of freeze-up can reverse the trend and cause the moisture content to decrease with increasing depth from the soil surface. Patterns can diverge substantially from these general trends, however.

Variations in the vertical distribution of moisture cause the average value for S_I (Equation (1)) to change from the corresponding value for a uniform distribution. That is, when the moisture content increases with depth the average S_I is smaller than the corresponding value for a uniform distribution. Conversely, a profile in which the moisture content decreases with depth causes the average S_I to be larger than the corresponding value for a uniform distribution.

In most frozen soils a vertical gradient in soil moisture (water + ice) does not usually result in a significant gradient in soil suction, especially at temperatures lower than about $-0{\cdot}5\,^{\circ}$C, because the unfrozen (liquid) water content changes slowly with decreasing temperature below this value. Thus, the primary effect of a decrease in S_I or θ_I (volumetric content−water + ice) on infiltration is due to the reduction in ice content, which increases the air-filled porosity, the soil permeability and the infiltration rate. An increase in S_I or θ_I has the opposite effect, since an increase in ice content causes the air-filled porosity, the soil permeability and the infiltration rate to decrease.

A series of trials was conducted with the numerical model HAWTS (Zhao *et al*., 1997; Zhao and Gray, 1997, 1999) to study the effects of the vertical distribution of soil moisture (water + ice) on cumulative infiltration. These tests assumed a linear variation in soil moisture that extended from the soil surface to various depths (see Figure 6a). Cumulative infiltration after 12 h was calculated for each distribution. This amount was normalized to the cumulative infiltration after 12 h assuming that the soil moisture was distributed uniformly. The ratio, referred to as the infiltration ratio, IR, is plotted against average difference in volumetric soil moisture (water + ice), V_a, i.e. Δ multiplied by the depth of wetting. Negative values of V_a represent a profile in which the moisture content is increasing with depth and positive values of V_a represent a profile in which soil moisture content decreases with increasing depth.

The data in Figure 6b show that the infiltration ratio IR is sensitive to the vertical distribution of soil moisture. Drier soils near the surface enhance infiltration and produce values of $IR > 1$. Conversely, wetter soils near the surface suppress infiltration and $IR < 1$. The association between IR and V_a is described by the expression:

$$IR = e^{-0{\cdot}35 V_a} \tag{5}$$

which has a coefficient of determination, $r^2 = 0{\cdot}95$. Other tests showed that the curve is independent of the other variables that comprise the parametric equation (Equation (1)).

These findings confirm the importance of the vertical distribution of soil moisture on infiltration into frozen soils. Large discrepancies among measured infiltration and values estimated by Equation (1) can be expected if the effect is not accounted for in the calculations.

Surface saturation, S_0

The parametric equation (Equation (1)) shows INF $\propto S_0^{2{\cdot}92}$, which indicates that infiltration is strongly influenced by surface saturation. In these regards, it is noted that this proportionality is valid only for unsaturated conditions (i.e. $S_0 \leq 1{\cdot}0$) and is due primarily to the nonlinearity of the relationship between soil suction and soil moisture. For a saturated surface or when water is ponded, infiltration increases more slowly with increasing hydraulic head.

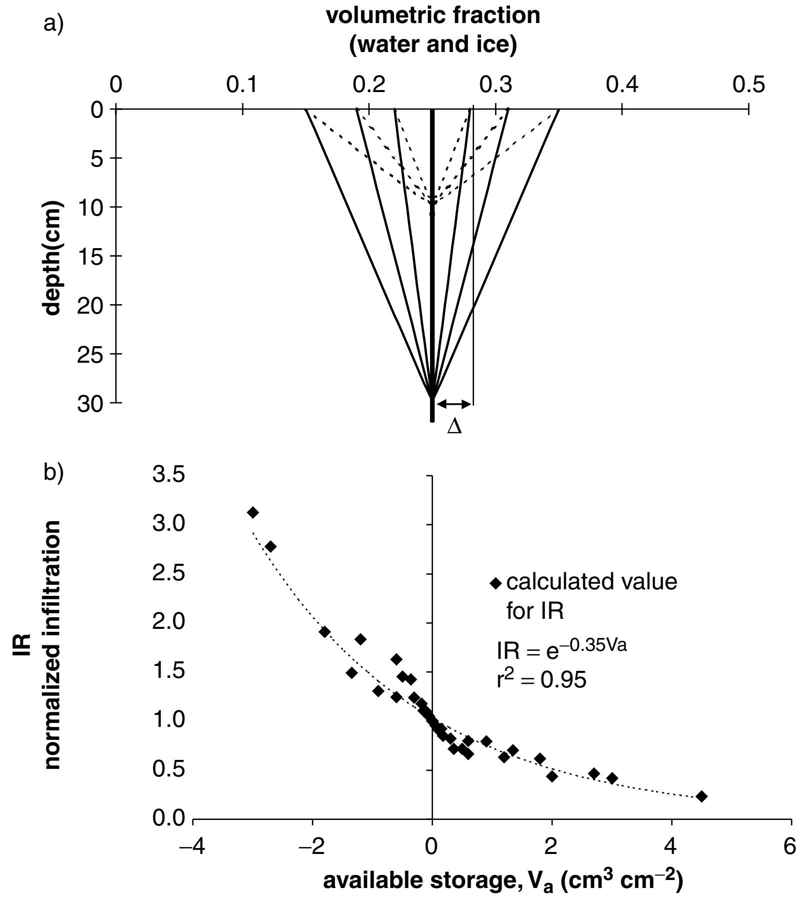

Figure 6. (a) Assumed linear variation in soil moisture extending from the soil surface to various depths. (b) Infiltration ratio, IR, plotted against the available storage, V_a (i.e. the average difference in volumetric soil moisture (water + ice), Δ, multiplied by the depth of wetting). Negative values of V_a represent a profile in which the moisture content is increasing with depth and positive values of V_a represent a profile in which soil moisture content decreases with increasing depth

Surface saturation during snow ablation is affected by the rate that meltwater is released by a snowcover, the infiltration rate of the frozen ground and the movement of meltwater laterally on the soil surface. Because reliable techniques for measuring S_0 have not been developed, it is suggested (based on field experience and limited measurements) that $S_0 = 0{\cdot}75 \rightarrow 1{\cdot}00$. In most situations the infiltration rate of a frozen soil is low and a value of $S_0 \cong 1$ should be assumed, especially under conditions when a snowcover ablates rapidly. An average value for $S_0 < 1$ may be experienced under slow ablation, when there are large fluctuations in the melt rate and where the ground surface is highly permeable. Fine-tuning of the parameter to simulate the effects of cycling in the melt rate might be achieved by varying the parameter according to the melt rate. Where this is attempted, the volume of infiltration calculated by the equation should be the same as the amount determined by an average value of S_0 over the period in the parametric equation (Equation (1)).

Initial saturation (water + ice), S_I

An inverse relationship between infiltration and moisture/ice content in frozen soils has been postulated by many investigators (e.g. Willis *et al.*, 1961; Kuzik and Bezmenov, 1963; Komarov and Makarova, 1973; Steenhuis *et al.*, 1977; Kane and Stein, 1983; Granger *et al.*, 1984; Burn, 1990). As the initial saturation

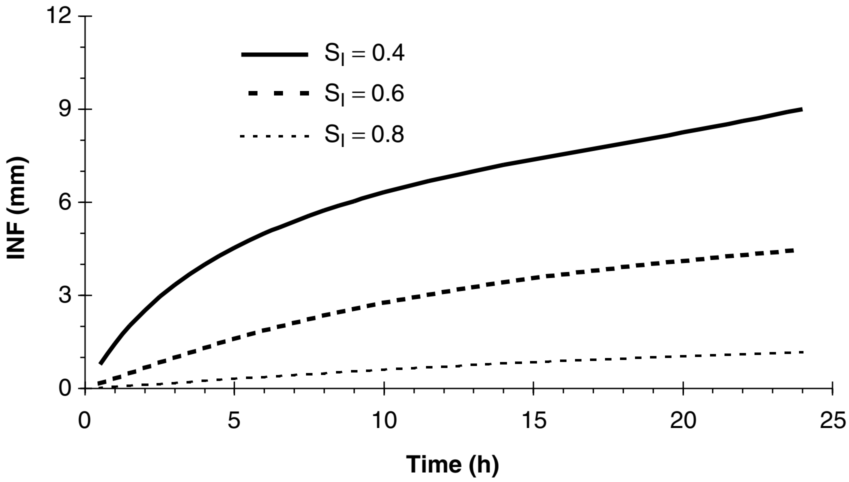

Figure 7. Variation in cumulative infiltration, INF, with time into a frozen, silty clay soil with $S_I = 0{\cdot}4$, $0{\cdot}6$, $0{\cdot}8$ and $T_I = -6\,^\circ C$, $S_0 = 0{\cdot}75$. Infiltration at hour 12 is 6·8 mm, 3·1 mm and 0·7 mm for $S_I = 0{\cdot}4$, $0{\cdot}6$, $0{\cdot}8$ respectively

increases, the capillary pressure gradient decreases and the ice content increases, which causes infiltration to decrease. Figure 7 plots cumulative infiltration with time into a silty clay soil at three levels of initial saturation: $S_I = 0{\cdot}40$, $S_I = 0{\cdot}60$ and $S_I = 0{\cdot}80$ at an initial temperature $T_I = -6\,^\circ C$ and $S_0 = 0{\cdot}75$. Infiltration at 12 h is 6·8 mm, 3·1 mm and 0·7 mm for $S_I = 0{\cdot}40$, $0{\cdot}60$ and $0{\cdot}80$ respectively. That is, a twofold increase in soil moisture decreased infiltration by a factor of about 10.

The discussions of initial saturation and the vertical distribution of soil moisture at the start of infiltration indicate that these factors have a dominant effect on the process in frozen soils. This paper suggests that the microscale of infiltration is set by the spatial variability of soil moisture.

Initial soil temperature, T_I

Komarov and Makarova (1973), Steenhuis *et al.* (1977) and Granger *et al.* (1984) suggest that the effects of soil temperature on snowmelt infiltration into frozen soils may be secondary to those of other parameters, such as the soil moisture content. This conclusion is supported by the parametric equation, which shows INF $\propto (273{\cdot}15 - T_I/273{\cdot}15)^{-0{\cdot}45}$ during quasi-steady state flow. Likewise, the infiltration rate, $\mathrm{dINF}/\mathrm{d}T_I$, changes slowly with decreasing initial soil temperature.

Figure 8 plots the variation in cumulative infiltration with time for three levels of initial temperature and constant initial saturation. The data show that the higher the initial temperature, the larger the cumulative infiltration. The decrease in infiltration with decreasing initial temperature is primarily due to the increase in impedance (decrease in permeability) due to the increase in ice content. For the extreme case in which the effects on impedance are ignored, INF at $t = 24$ h and $T_I = -4$, -6 and $-8\,^\circ C$, agreed within 6%.

Because freezing temperature has a relatively small effect on frozen soil infiltrability it is suggested that spatial variations in soil temperature be ignored in scaling the infiltrability of *Limited* soils. The exceptions are cases where the initial temperature affects the areal extent of frozen soils, the depth of soil freezing and the development of an impeding ice layer at the ground surface.

Infiltration opportunity time, t_0

For ripe snowcovers the total infiltration opportunity time, t_0, is approximately equal to the time required to melt a snowcover, t. Assuming continuous melting and small storages and evaporation:

$$t_0 \cong t = \frac{SWE}{M} \tag{6}$$

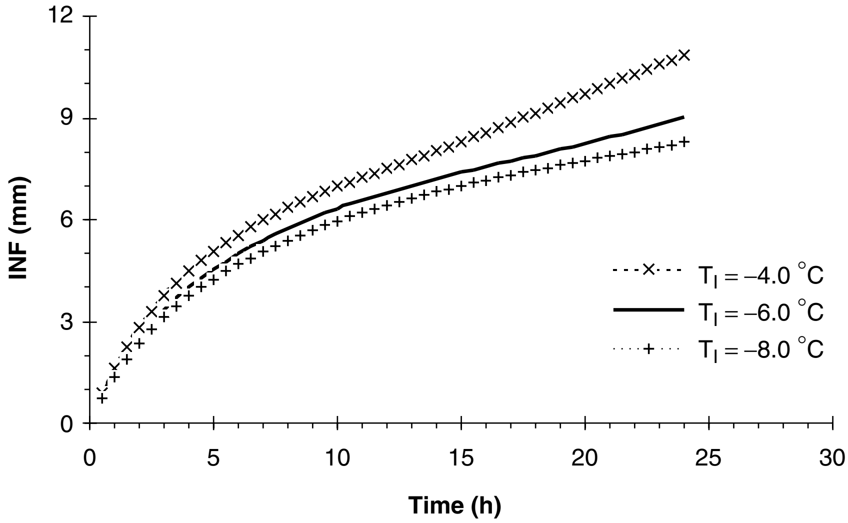

Figure 8. Variation in cumulative infiltration, INF, with time into a frozen, silty clay soil with $T_I = -4\,°C$, $-6\,°C$ and $-8\,°C$ at $S_I = 0{\cdot}4$, $S_0 = 0{\cdot}75$. Infiltration at hour 12 is 7·5 mm, 6·7 mm and 6·4 mm for $T_I = -4\,°C$, $-6\,°C$ and $-8\,°C$ respectively

where SWE is the snow water equivalent and M is the average melt rate during time t. In natural, unripe snowcovers the direct association among variables suggested by Equation (6) will change because of storage and refreezing of meltwater within a snowcover. An indication of the association between seasonal infiltration, INF, SWE and t_0 in frozen prairie soils is obtained by equating the expression (Gray *et al.*, 1985):

$$\mathrm{INF} = 5(1 - S_I)\mathrm{SWE}^{0{\cdot}584} \tag{7}$$

with the parametric equation (Equation (1)) that shows:

$$\mathrm{INF} \propto \mathrm{SWE}^{0{\cdot}584} \propto (t_0)^{0{\cdot}44} \tag{8}$$

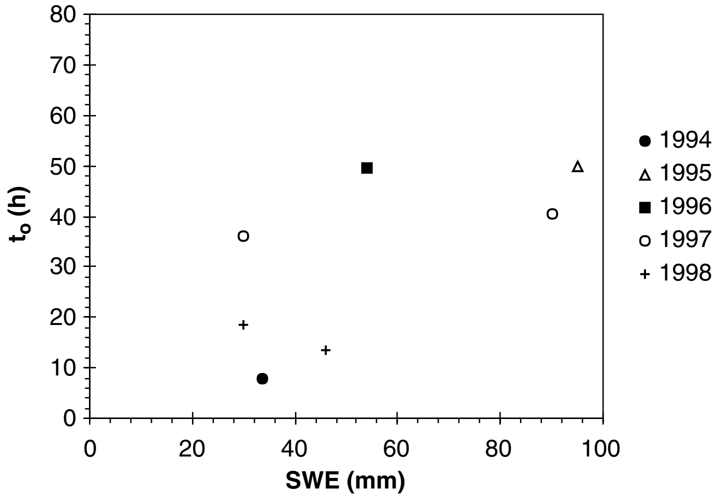

Figure 9. Variation in infiltration opportunity time, t_0, with snow water equivalent, SWE, at a cleared site in a boreal forest, 1994–98

and

$$t_0 \propto \mathrm{SWE}^{1\cdot33} \tag{9}$$

The proportionalities given as Equation (8) suggest that measurement errors in SWE have a small effect on the amount of infiltration, especially in deep snowcovers. Zhao and Gray (1997) analysed the relationship between t_0 and SWE for prairie snowcovers using measured values for INF, S_I and T_I and $S_0 = 0{\cdot}9$ in Equation (1) to calculate the opportunity time. They found that plots of 'seasonal' infiltration opportunity time versus SWE at the start of ablation demonstrated a strong linear trend. Open sites in a boreal forest exhibit similar organized trends between years when the melt rate does not vary widely (see Figure 9). However, in forests the spatial and temporal associations of t_0 and SWE are highly variable because of the effects of vegetation on SWE and the ablation rate and the covariance between the ablation rate and depth of accumulated snow (see Figure 10) (Faria, 1998: Faria *et al*., 2000).

The discussions above suggest SWE can be used to index the infiltration opportunity time. For snow conditions where the covariance between snow depth, d, and snow density, ρ_s is known, SWE can be

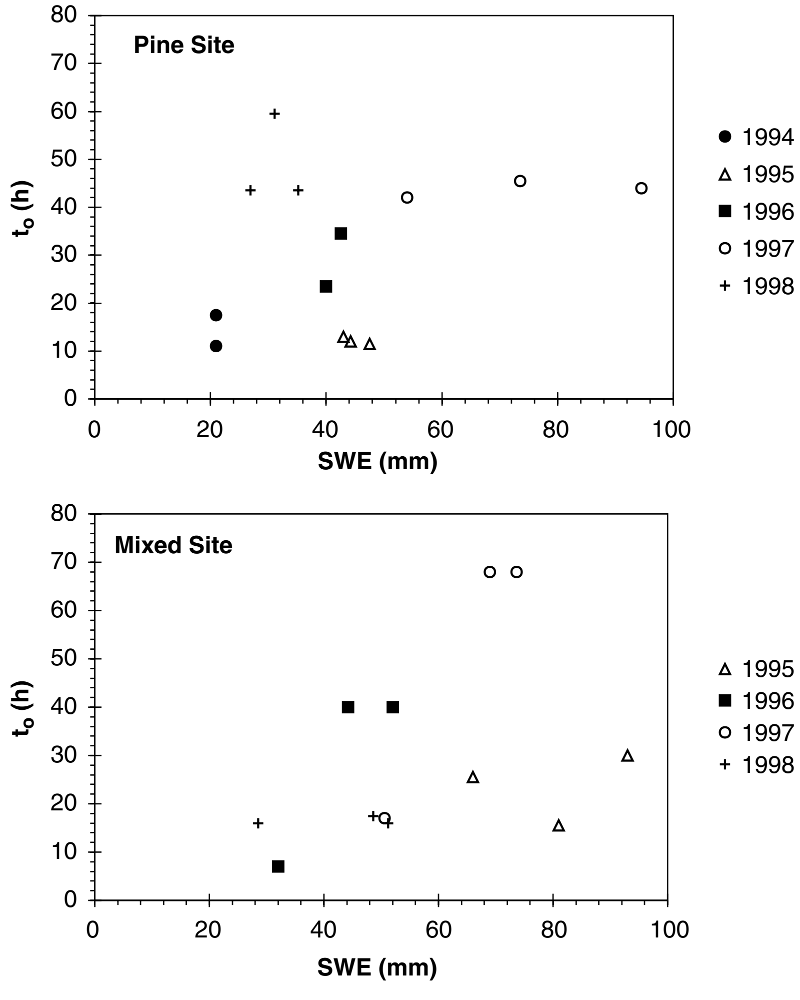

Figure 10. Variation in infiltration opportunity time, t_0, with snow water equivalent, SWE, at pine and mixed vegetation sites in a boreal forest, 1994–98. Leaf area index (LAI) measurements at pine site ranged from 1·99 to 2·20 m^2/m^{-2}. At the mixed site LAI exhibits larger variability, from 1·72 to 2·90 m^2/m^{-2}

replaced by d in the correlation and the spatial variability in infiltration scaled according to variations in snow depth. It follows that the infiltrability scale should not be larger than the scales used to describe either the spatial variations of snow water equivalent and/or depth or the covariances between snow water equivalent and ablation rate.

SCALE

General

The material above sets out general criteria for scaling frozen soil infiltrability in studies of snow ablation. The infiltration scale should be consistent with the scale(s) describing the spatial variations of other primary factors affecting the process, namely ground temperature, snow water equivalent and snowmelt. If a hierarchical order of scales is hypothesized, the minimum scale of the parameter of highest order establishes the maximum scale of the parameter of lowest order or vice versa. In as much as:

1. Snow water equivalent has a greater effect on infiltration than ground temperature and the spatial variations of SWE are normally greater (in a relative sense) than divergences of soil temperature.
2. The effects of freezing temperature on infiltration are accounted for in the infiltration model.
3. A large amount of knowledge exists on the association between snow accumulation patterns and land use, vegetation and topographic factors (Pomeroy and Gray, 1995; Pomeroy *et al*., 1998, see Table I).
4. First generation models for generating a snow field from point measurements are currently under development and testing.
5. The possibilities (likelihood) of strong linkages between SWE and soil temperature, melt rate and soil moisture (water + ice), especially among land units with similar land use and topography.

it is suggested that the maximum scale for infiltration not exceed the minimum scale required to describe the spatial distribution of the snow water equivalent.

Application

General. Normally, the identification and division of an areal unit into *Restricted, Limited* and *Unlimited* parts would be done at freeze-up based on precipitation and evaporation history and field observations. However, mechanisms should be built into the procedure so that the boundaries of an infiltrability class can be modified to accommodate changes to infiltration properties that may occur during winter. As a rule the boundaries of *Unlimited* soils will not change from those determined in the fall of the previous year. Conversely, mid-winter melt events and refreezing of meltwater at the soil surface may cause *Limited* soils to become *Restricted.* Having established the areal extents of *Unlimited* and *Restricted* soils, the area of *Limited* soils is taken as the residual of the total snow-covered area. Determination of frozen soil infiltration for *Limited* soils involves the application of Equations (1) and (4) using average values of the parameters for the landscape type. Subsequent division of the area of *Limited* soils within a landscape unit (CV_{SWE} = constant) to accommodate special mesoscale applications should be based on the spatial variability of the mean initial saturation, $\overline{S_I}$, whose value may vary with major changes in terrain variables, in vegetative cover and possibly in soil texture—since $\overline{S_I}$ is affected by the soil water holding properties. Conversely, upscaling may be accomplished by grouping landscape units with common (CV_{SWE}, $\overline{S_I}$) and where covariance of CV_{SWE} and $\overline{S_I}$ can be established.

Scaling snow water distribution and frozen soil infiltration at smaller scales usually are necessary in studies of snow ablation where the energy supplied by local advection is important to snowmelt and information on the distribution of soil water recharge is required. Shook and Gray (1996) discuss the scale of a snowcover required to incorporate the semifractal spatial distribution of SWE so as to maintain the integrity of the soil and snow patch geometries that develop during ablation. They show that the scaling length is related to the macroscopic variability of the underlying topography. On flat land, the length is about 30 m, independent of

Table I. Representative values for the coefficient of variation of water equivalent of snowcovers, CV_{SWE}, on various landscapes in prairie, arctic and boreal forest environments in late winter (after Pomeroy *et al*. 1998 (An evaluation of snow processes for land surface modelling, *Hydrological Processes* **12**, 2339–2367 Pomeroy JW, Gray DM, Toth B, Essery RHL, Pietroniro A, Hedstrom N. 1998. © John Wiley & Sons Limited. Reproduced with permission.))

Region	Land use and Vegetation	Landform	CV_{SWE}
Prairies	Fallow	Flat plains; slightly to moderately rolling topography with gentle slopes	0·47
		Bottom (bed) of wide waterways; large sloughs and depressions	0·30
		Crests of hills, knolls and ridges	0·58
	Stubble	All landforms	0·33
	Pasture	Flat plains; bottom (bed) of wide waterways; large sloughs and depressions; slightly to moderately rolling topography with gentle slopes	0·41
		Crests of hills, knolls and ridges	0·51
		Lee of abrupt, sharp slopes	0·57
	Scattered brush	Bottom (bed) of waterways, e.g., gullies; sloughs and depressions	0·42
		Lee of abrupt sharp, slopes	0·52
	Treed farmyards		0·50
Arctic	Tundra	Flat plains, upland plateaus, slight to moderately rolling topography	0·31
		Valley bottoms	0·28
		Valley sides (drifts where slopes greater than 9°)	0·34
	Shrub tundra	Flat plains, slight to moderately rolling topography	0·22
		Valley bottoms	0·16
		Valley sides (drifts where slopes greater than 9°)	0·18
	Sparse forest tundra	Exposed hillside and forest edge	0·21
		Sheltered	0·11
Boreal forest	Black spruce	All Landforms	0·14
	Mature jack pine		0·10
	Mixed aspen–white spruce		0·05
	Young jack pine		0·14
	Recent clear-cut		0·07
	Recent burn		0·04

surface condition (wheat, stubble or fallow). The distance increases on hummocky terrain (50 m) and across wide valley floors (500 m). It is concluded that the variability in large-scale topography is the primary cause for the increase in distance of the transition of snow depth from fractal to random structure. Recent work by Bárdossy and Lehmann (1998) on the spatial distribution of soil moisture over a small (6·3 km^2), intensely farmed, hilly watershed in Germany found a correlation length of similar order of magnitude (about 300 m) for moisture measurements in the soil depth increment 0–15 cm.

Flow chart. A flow chart showing the implementation of an algorithm, which is based on the discussions above, to determine cumulative areal infiltration on a hydrological response unit is shown in Figure 11. The definitions of symbols used in the figure are listed in Table II. The first discretization partitions the response unit into landscape units according to the distribution of snow water. The infiltration model is then applied to each individual landscape unit that is frozen and receives water from melting snow.

- The areas of the landscape having *Unlimited, Limited or Restricted* infiltrability are identified.
- If the land unit is *Restricted*, infiltration is set to zero.

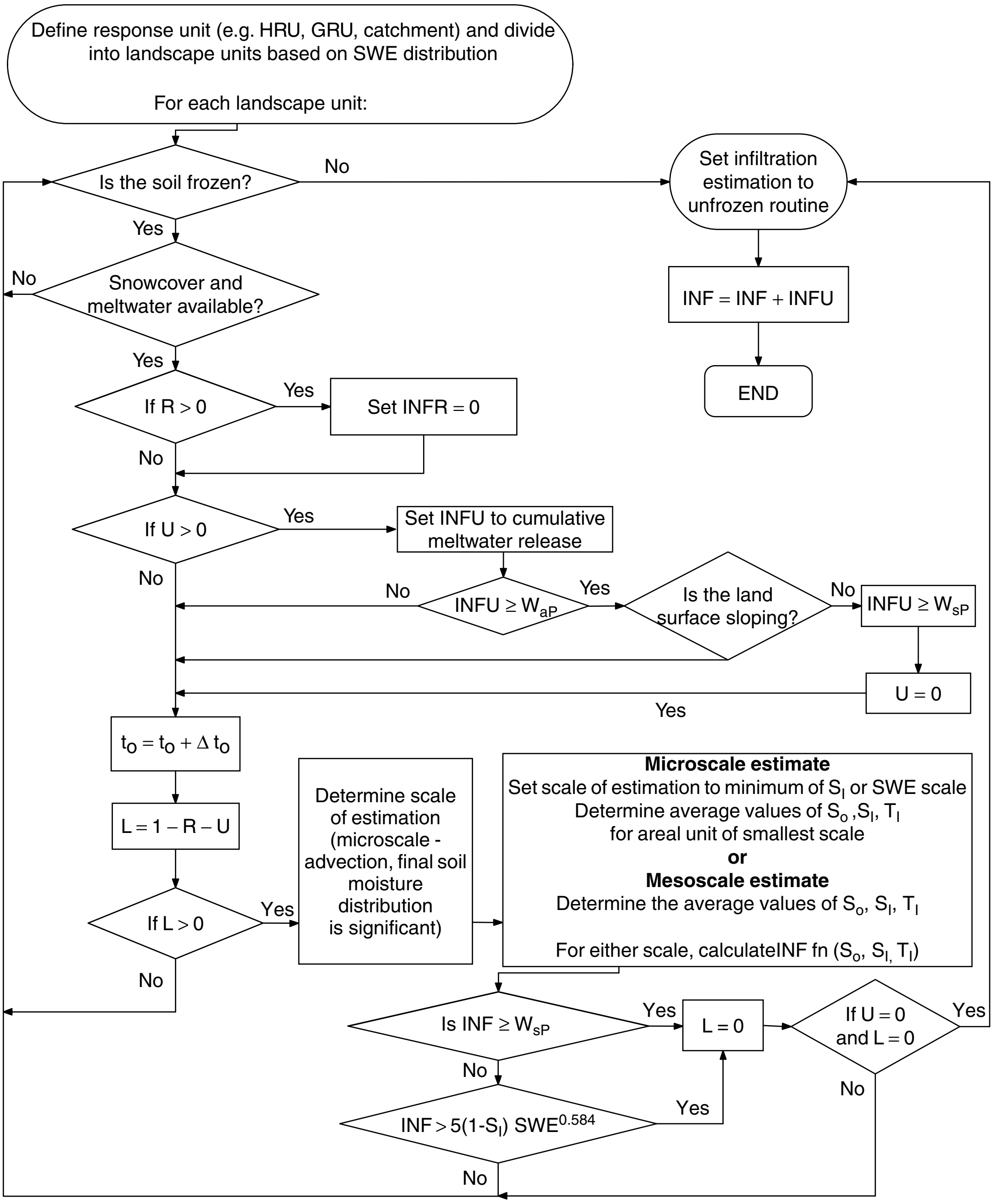

Figure 11. Flowchart of an algorithm for estimating cumulative areal snowmelt infiltration into frozen soils

- If the land unit is *Unlimited*, infiltration is set to the cumulative meltwater release. On flat land the total amount time is constrained by the water holding capacity of the frozen soil, W_{sP}.
- All other portions of the landscape unit have *Limited* infiltrability and infiltration is controlled by the average surface saturation, S_0, the average initial soil saturation, S_I, and the average initial temperature, T_I, of the 0–40 cm soil layer (Equation (1)) and constrained by the available water storage, W_{sP}.

Table II. Definitions of symbols used in Figure 11

Symbol	Definition
INF	cumulative infiltration into soils in *Limited* class
INFR	cumulative infiltration into soils in *Restricted* class
INFU	cumulative infiltration into soils in *Unlimited* class
L	fraction of landscape unit in the *Limited* class
R	fraction of landscape unit in the *Restricted* class
S_0	average surface saturation during time step
S_I	average initial soil moisture of 0–40 cm soil layer at start of infiltration
SWE	snow water equivalent
t_0	cumulative infiltration opportunity time
T_I	average initial soil temperature of 0–40 cm soil layer at start of infiltration
U	fraction of landscape unit in the *Unlimited* class
W_sP	water holding capacity of frozen soil

- If the study is a mesoscale application, infiltration is estimated using average values of S_0, S_I and T_I for the landscape.
- If the study is a microscale application, infiltration is determined at a spatial scale set by either the spatial distribution of S_I or the scale of the spatial distribution of snow water, whichever is smallest. At the smaller scale, infiltration is estimated by Equation (1) using average values of S_I, S_0 and T_I of the smaller unit.
- If cumulative infiltration up to the time exceeds the available water holding capacity the soil is considered thawed and the infiltration estimation set to the unfrozen routine.

SUMMARY AND CONCLUSIONS

A methodology based on findings of theory, experiment and field observation for scaling the infiltrability of frozen soils in snow ablation studies is described. Frozen soils are grouped into the three general classes according to their surface entry condition for meltwater as: *Unlimited*—high infiltrability, *Restricted*—low infiltrability and *Limited*—infiltration is governed by soil physical properties. For *Limited* soils, cumulative infiltration with time is estimated from surface saturation, initial soil saturation (water + ice), initial soil temperature and infiltration opportunity time. Estimates of cumulative infiltration into *Unlimited* and *Limited* soils are constrained by the available water storage capacity (air-filled porosity).

In applying the infiltration model to estimate areal snowmelt infiltration, it is recommended that the maximum scale for infiltration not exceed the minimum scale required to describe the spatial distribution of the snow water equivalent. Average values of the soil variables for the landscape type or snow class describing them are used to estimate infiltration into *Limited* soils and storages.

Scaling frozen soil infiltration at small scales (microscale) in studies of snow ablation requires information on the areal depletion of snowcover and the spatial distribution of soil water (water + ice). Further research on the association(s) between the spatial distributions of snow water and soil water is needed.

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