JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 107, NO. D24, 4767, doi:10.1029/2001JD001280, 2002

An extension of the force-restore method to estimating soil temperature at depth and evaluation for frozen soils under snow

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Received 10 September 2001; revised 27 February 2002; accepted 20 June 2002; published 20 December 2002.

[1] The force-restore method (FRM) was originally developed for estimating diurnal fluctuations in the ground surface temperature. Because of its relatively simple parameterization, it is commonly applied in meteorological and other models for this purpose. Its application to the calculation of deeper soil temperatures, to frozen soils, and to soils under snow covers has heretofore not been possible. This study demonstrates an extension of the FRM that permits accurate estimates of seasonal variation in mean daily deep soil temperature. The extended FRM is shown to provide a lower boundary condition for the heat conduction method, permitting a combination of the two approaches that avoids some limitations of each. The combined approach provides representations of the mean daily soil temperature, soil temperature at depth in frozen soils, and ground surface temperature under a snow cover. Diurnal variations can also be calculated. The extended method and combined approaches are tested using field site measurements collected in cold weather periods in Saskatchewan, Canada, and are found to provide a reasonable representation of measurements. *INDEX TERMS:* 3322 Meteorology and Atmospheric Dynamics: Land/atmosphere interactions; 3337 Meteorology and Atmospheric Dynamics: Numerical modeling and data assimilation; 1823 Hydrology: Frozen ground; 1878 Hydrology: Water/energy interactions; 1863 Hydrology: Snow and ice (1827); *KEYWORDS:* soil temperature, force-restore method, frozen soil

Citation: Hirota, T., J. W. Pomeroy, R. J. Granger, and C. P. Maule, An extension of the force-restore method to estimating soil temperature at depth and evaluation for frozen soils under snow, *J. Geophys. Res.*, *107*(D24), 4767, doi:10.1029/2001JD001280, 2002.

1. Introduction

[2] Ground surface temperature is an extremely important term for calculating radiative and turbulent heat fluxes and the exchange of water vapor and other gases between land surfaces and the atmosphere. Subsurface soil temperatures and their vertical gradients exert a strong control on conductive heat flux, soil heat storage and, by influencing root temperatures, plant growth rates, primary productivity, decomposition and evapotranspiration. The presence of frozen soil has a dramatic effect in decreasing infiltration and increasing runoff from surface water supplied by precipitation or snowmelt. Temperature regimes, frozen soil depths, and vertical temperature gradients under snow covers are important for estimates of over winter desiccation of frozen soils, soil nitrification or denitrification and the production and release of greenhouse gases from soils [*Gray et al.*, 1985; *Van Bochove et al.*, 2001]. There is increasing realization that the method and hence accuracy of calculating soil temperature have extremely important implications for the timing and rate of snowmelt in land surface schemes [*Lynch et al.*, 1998]. Techniques that would improve estimates of soil temperature for winter conditions could find application in the solution of a variety of meteorological, hydrological, soil ecological and agricultural problems. Two methods are commonly used to calculate soil temperature based on an energy balance at the surface, the heat conduction equation (HCE) [e.g., *Campbell*, 1985] and the force-restore method (FRM) [e.g., *Bhumralkar*, 1975].

[3] Solving the HCE provides a robust method of calculation for both ground surface and soil temperature [e.g., *Campbell*, 1985]. However, because of sharp vertical gradients that commonly develop in soil temperatures near the

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surface, this method requires high-resolution, multisoil layer calculations to provide accurate, stable solutions. The HCE also requires the assignment of an initial vertical profile of soil temperature and lower boundary condition for this profile or the soil heat flux, and parameterizations of soil thermometric relations. The required parameterizations depend on the timescale of application. For diurnal soil temperature calculations, the depth of the lower boundary condition can be taken at 0.3–0.5 m, because soil temperatures at these depths change only slightly over a day and can be considered constant. For daily to seasonal calculations, there can be a marked temperature regime at these depths and therefore the lower boundary condition needs to be at depths from several meters to tens of meters. Numerical solutions of the HCE over these time periods can drift and need to be reinitialized periodically to observed temperature profiles in order to preserve accuracy. These values are not easy to estimate and usually require observations (e.g., daily soil temperature profiles). Because of these drawbacks, the HCE has limitations in its application as an effective operational method to calculate ground surface and soil temperature. For instance, the ERA40 surface scheme of the ECMWF model uses distinct ground and snow HCEs coupled to canopy and atmospheric convection, via resistances [*Van den Hurk et al.*, 2000]. When evaluated against the BOREAS data set from a northern Saskatchewan forest, the ERA40 simulation (including effects of snow) was up to 15°C colder in midwinter than measurements in the equivalent shallow soil layer just under the snowpack. At the FIFE site in a Kansas grassland (much warmer winter than Saskatchewan) similar comparisons showed that ERA40 simulations were about 5°C colder than measurements in shallow soil in midwinter [*Van den Hurk et al.*, 2000].

[4] The FRM is an alternative approach, developed to estimate the ground surface temperature [e.g., *Bhumralkar*, 1975; *Deardorff*, 1978; *Lin*, 1980; *Dickinson*, 1988]. This method has been incorporated in many numerical hydrological and atmospheric models [e.g., *Noihan and Planton*, 1989; *Dickinson et al.*, 1993; *Kimura*, 1994; *Blackadar*, 1997]. *Hu and Isram* [1995] showed that the FRM could provide accurate estimates of both ground surface and upper soil temperatures by minimizing the error between the analytical solution from the FRM and from that of the HCE under diurnal forcing. *Hirota et al.* [1995] demonstrated and tested with field measurements in Japan, an extension of the FRM to estimate the seasonal variation in daily mean soil temperature of shallow (upper) soil layers. However, these versions of the FRM did not consider estimating deep soil temperature, nor have they been fully tested under snow cover and for frozen soils. *Tilley and Lynch* [1998] suggested that the FRM would be too unresponsive for simulation of soil temperature in permafrost regions. It is doubtful that existing FRM formulations can accurately represent the surface temperature of soil under a snow cover as they assume a strong diurnal "forcing" at the surface. As noted by *Slater et al.* [2001], insulation of soil from air by snow cover will strongly dampen diurnal temperature fluctuations at the ground surface and may violate the force-restore assumption. The prospects of successfully applying an unmodified FRM approach to cold, snow-covered regions are therefore questionable. The PILPS 2(d) land surface scheme intercomparison group has called for new approaches involving multilayer models that address conduction but with the benefits of the FRM [*Slater et al.*, 2001].

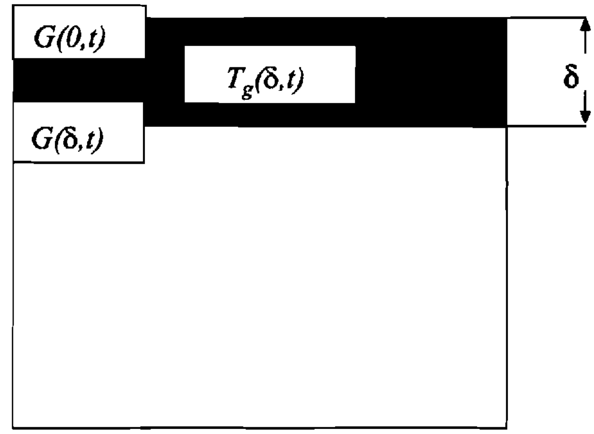

Figure 1. Schematic diagram of the FRM for a soil surface layer.

[5] This study proposes a new and simple method for estimating deep soil temperature using a modification of the FRM. The FRM can predict diurnal variations in ground surface temperature if appropriate boundary conditions can be specified. Here, extended methods for seasonal variations in mean daily deep soil temperature are shown. The new method also can be applied to determine lower boundary condition of soil temperature to estimate ground surface temperature or vertical soil temperature profile. It can be applied for not only mean daily value but also diurnal variations. This application of the FRM is also effective for estimating daily boundary conditions and estimating soil temperature profile for frozen and unfrozen soil under snow. The results of an unmodified FRM, the modified FRM, and the HCE are compared to measurements of soil temperature regimes over a winter near Saskatoon, Saskatchewan, Canada.

2. Extension of the FRM

2.1. Review of Method

[6] *Bhumralkar* [1975], *Lin* [1980], and *Hu and Isram* [1995] outlined the original FRM and provide the basis for its description below. Assuming a homogeneous soil with vertical heat flow, conduction of heat is related to the vertical temperature gradient and its evolution over time:

$$\frac{\partial T(z,t)}{\partial t} = \frac{\lambda}{c}\frac{\partial^2 T(z,t)}{\partial z^2} \quad (1)$$

where $T(z, t)$ is the soil temperature over some vertical coordinate, z, and time t, λ is the soil thermal conductivity, and c is the volumetric heat capacity. Equation (1) is the classical HCE. Temperature at the ground surface boundary is considered subject to a sinusoidal fluctuation:

$$T(0,t) = \overline{T} + \Delta T_0 \sin(\omega t) \quad (2)$$

where $\overline{T}$ is the mean ground surface temperature (daily or annual), ΔT_0 is the daily or annual temperature amplitude at

Table 1. Example of Annual Mean Soil Temperature (°C) at Different Depths

	Location			
	Sapporo (Japan)[a]	Tokyo (Japan)[a]	Naha (Japan)[a]	Saskatoon (Canada)[b]
Depth (m)	43°03'N, 141°20'E, 16.9 m asl	35°41'N, 139°46'E, 6.5 m asl	26°15'N, 127°41'E, 34.8 m asl	52°09'N, 106°36'W, 497 m asl
0.0	8.7	15.5	23.2	–
0.1	8.7	15.4	22.9	–
0.2	9.4	15.6	22.9	5.6
0.5	9.4	16.0	23.3	6.0
1.0	9.3	16.1	23.6	6.1
3.0	9.0	16.4	23.3	5.9

[a]*Ministry of Agricultural and Forestry and Fishers and the Meteorological Agency in Japan, Soil Temperature Databook in Japan* [1982].
[b]Data from Saskatchewan Research Council Climate Reference Station.

the surface, and ω is the frequency of oscillation equal to $2\pi/\tau$. Using equation (2), the solution for soil temperature at depth becomes

$$T(z,t) = \overline{T} + \Delta T_0\, e^{-z/D_a} \sin(\omega t - z/D_a) \tag{3}$$

where D_a is the damping depth (m) of surface temperature fluctuations, found as

$$D_a = (2a/\omega)^{0.5}, \tag{4}$$

α is the soil thermal diffusivity, found as $\alpha = \lambda/c$, $\omega = 2\pi/\tau$ and τ is the period of temperature fluctuation calculation (day or year).

[7] The vertical conductive heat flux in a soil, G, at depth z and time t is given by

$$G(z,t) = -\lambda \frac{\partial T(z,t)}{\partial z} \tag{5}$$

Combining equations (5) and (3) provides

$$G(z,t) = \Delta T_0 \left(\frac{\omega c \lambda}{2}\right)^{0.5} e^{-z/D_a} [\sin(\omega t - z/D_a) + \cos(\omega t - z/D_a)] \tag{6}$$

Furthermore, eliminating ΔT_0 from equation (6) by using equation (3) and its differential form with respect to t results in

$$G(z,t) = \left(\frac{\omega c \lambda}{2}\right)^{0.5} \left(\omega^{-1} \frac{\partial T(z,t)}{\partial t} + T(z,t) - \overline{T}\right) \tag{7}$$

which is a differential form of the soil heat flux with respect to time (t). Equations (5) and (7) provide the soil heat flux as a function of the partial derivatives of temperature with respect to time and depth [*Bhumralkar*, 1975].

2.2. Application to the Soil Surface Layer

[8] Considering a soil surface layer of thickness δ below the ground surface, as shown in Figure 1, then the rate of temperature change over time for this layer is given by

$$c \frac{\partial T_g(\delta,t)}{\partial t} = -\left(\frac{G(\delta,t) - G(0,t)}{\delta}\right) \tag{8}$$

T_g, the ground surface temperature, is defined as

$$T_g(\delta,t) = \frac{1}{\delta} \int_0^{\delta} T(z,t)dz \tag{9}$$

Assuming that $T(\delta, t) \approx T_g(\delta, t)$, and combining equations (8) and (7),we obtain

$$C_1(\delta) \frac{\partial T_g(\delta,t)}{\partial t} = \frac{2}{cD_a} G(0,t) - \frac{2\pi}{\tau}(T_g(\delta,t) - \overline{T}) \tag{10}$$

Here, $C_1(\delta) = (1 + 2\delta/D_a)$. This is equivalent to *Bhumralkar*'s [1975] original version of the FRM. *Lin* [1980] argued that the assumption ($T(\delta, t) \approx T_g(\delta, t)$) is not realistic because large temperature gradients can exist near the surface. He proposed for diurnal estimates that $C_1(\delta) = (1 + \delta/D_a)$. *Hu and Isram* [1995] developed a new C_1 function to minimize the difference between the analytical solution of the FRM and the full HCE in response to a single periodic forcing. Their polynomial approximation is

$$C_1(\delta) = 1 + 0.943(\delta/D_a) + 0.223(\delta/D_a)^2 + 1.68 \times 10^{-2}(\delta/D_a)^3 - 5.27 \times 10^{-3}(\delta/D_a)^4 \tag{11}$$

For case of $\delta \to 0$ in equation (10),

$$\frac{\partial T(0,t)}{\partial t} = \frac{2}{cD_a} G(0,t) - \frac{2\pi}{\tau}(T(0,t) - \overline{T}) \tag{12}$$

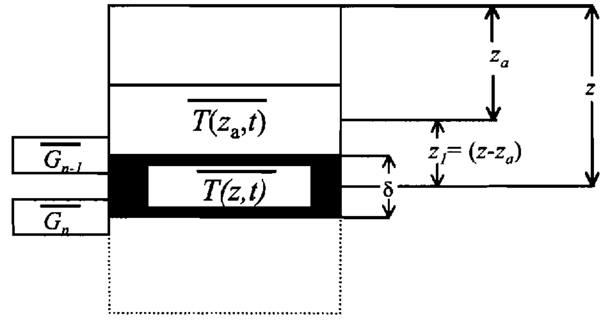

Figure 2. Schematic diagram of the model for estimating deep soil temperature of daily mean value.

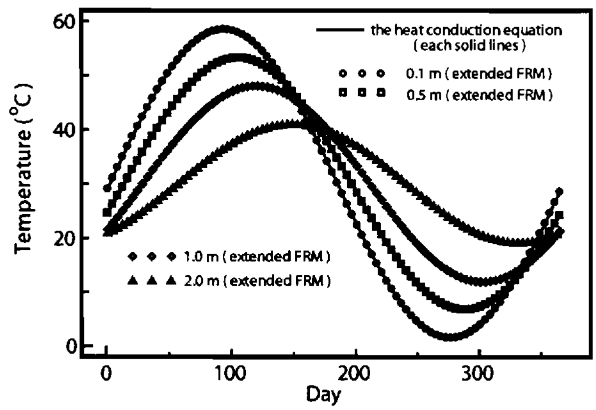

Figure 3. Comparison of the mean daily soil temperatures between the analytical solution of the HCE and the extended FRM (z_1 = 0.05 m).

This, in the same form as the original, is a FRM of ground surface temperature [*Deardorff*, 1978] for which $T(0, t) = T_g(0, t)$.

2.3. Extension to Mean Daily Soil Temperature

[9] The FRM can be applied from equation (12) to estimate variations in ground surface temperature, however its extension to calculation of soil temperature has been restricted for several reasons. It has been maintained that determining the daily mean ground surface temperature is problematic in that a value of $\overline{T}$ is required before solving for the diurnal variations of soil temperature using the FRM [*Deardorff*, 1978; *Dickinson*, 1988; *Kimura*, 1994; *Mihailovic et al.*, 1999]. In addition, $\overline{T}$ is required at depth to provide a lower boundary condition for diurnal calculations. The value of $\overline{T}$ may also have to respond to changing surface thickness, δ from day to day.

[10] Application to annual variations may be less of a problem as field measurements show that the mean annual soil temperature, $\overline{T_{ym}}$, is relatively invariant with depth (Table 1). Therefore $\overline{T_{ym}}$ can be treated as a constant for a location, and its value need not to be changed with changing δ. $\overline{T_{ym}}$ at a location can be estimated using well-tested simple empirical equations from the mean annual air temperature [e.g., *Arakawa and Higashi*, 1951; *Nishizawa*, 1992], permitting a relatively easy parameterization of the FRM for calculations of annual variation in mean daily soil temperature [*Hirota et al.*, 1995].

[11] The annual soil temperature variation damping depth D_a must also be estimated to use the FRM for annual variations. Annual period is 365 times the daily period. From equation (4), the D_a for annual variations is $\sqrt{365}$ times (about 19.1 times) the value of D_a for diurnal variations. From the second term of equation (3), the effects of soil thermal diffusivity on calculation of soil temperature decrease exponentially due to the value of D_a for annual variations is much larger than the value of D_a for diurnal variations. Hence, the effect of soil thermal diffusivity on the calculation of annual variations in the mean daily soil surface temperature is small, compared to the effect on diurnal variations [*Hirota et al.*, 1995]. For annual variations it is therefore unnecessary to accurately estimate the thermal diffusivity of soil.

[12] The deeper D_a for annual variations also means the thickness of the surface layer can be considered much deeper than for diurnal variations. For instance, the diurnal soil temperature change at 0.01 m depth is equivalent to the annual variation at about 0.2 m depth. Therefore, for annual variations a soil surface layer may be defined up to several tens of centimeters. These characteristics are important to estimating mean daily soil temperatures by FRM.

2.4. Extension to Mean Daily Soil Temperatures at Depth

[13] The rate of temperature change with time for an internal soil layer as shown in Figure 2 is given by

$$c\frac{\partial \overline{T(z,t)}}{\partial t} = -\left(\frac{\overline{G_n} - \overline{G_{n-1}}}{\delta}\right) \tag{13}$$

Combining equations (7) and (13) provides

$$C_1(\delta)\frac{\partial \overline{T(z,t)}}{\partial t} = \frac{2}{cD_a}\overline{G_{n-1}} - \frac{2\pi}{\tau_y}\left(\overline{T(z,t)} - \overline{T_{ym}}\right) \tag{14}$$

Here, $\overline{T(z,t)}$ is the daily mean soil temperature, τ_y is the annual period (365 days), $\overline{G_n}$ is the daily mean soil heat flux at the bottom boundary of the soil layer and $\overline{G_{n-1}}$ is the daily mean soil heat flux between an upper and internal soil layer; expressed as follows,

$$\overline{G_{n-1}} = -\lambda\frac{\partial \overline{T(z,t)}}{\partial z_1} \tag{15}$$

Combining equations (14) and (15) obtains

$$C_1(\delta)\frac{\partial \overline{T(z,t)}}{\partial t} = -\frac{2\lambda}{cD_a}\frac{\partial \overline{T(z,t)}}{\partial z_1} - \frac{2\pi}{\tau}\left(\overline{T(z,t)} - \overline{T_{ym}}\right) \tag{16}$$

where z_1 is the distance from upper soil depth to internal soil depth.

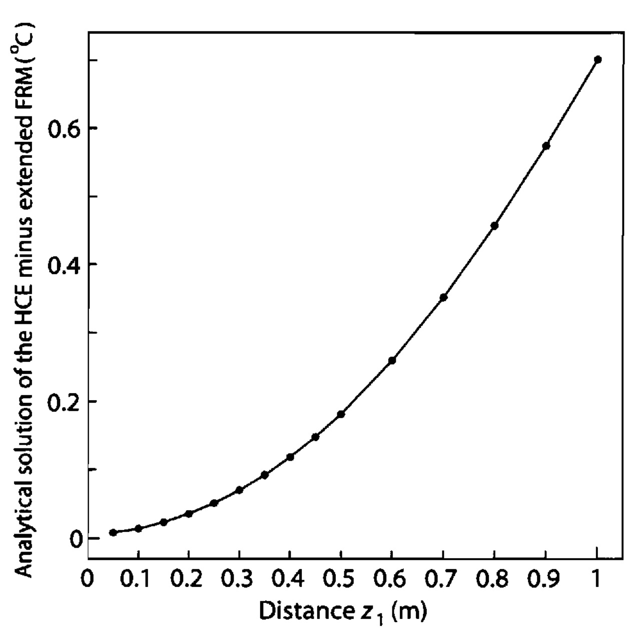

Figure 4. The relationship between distance z_1 and maximum temperature difference between the analytical solution of the HCE and the extended FRM for a calculation period of 365 days.

[14] Note that when solving the HCE, $\frac{\partial T}{\partial t} = \frac{\lambda}{c}\frac{\partial^2 T}{\partial z^2}$, for annual variations in soil temperature, it is necessary to set a lower boundary condition at several to several tens of meters depth: At the lower boundary, soil temperature is constant or the soil heat flux is zero. On the other hand, the extended FRM using equation (16) can calculate mean daily soil temperature $\overline{T(z,t)}$ from upper soil temperatures ($\overline{T(z_a,t)}$ in Figure 2) or $\overline{G_{n-1}}$ under given parameters (λ, c, D_a, and $\overline{T_{ym}}$). This means that it does not need to consider deeper soil conditions as inputs. Thus, equation (16) permits the derivation of flexible lower boundary conditions that can be provided to calculations such as the HCE. This detailed description is given in section 3.2.

3. Application of the Extended FRM

3.1. Comparison to the HCE Given Set Boundary Conditions

[15] The extended FRM for the mean daily soil temperature calculation was compared to an analytical solution to the HCE (equation (3)). This comparison used a sinusoidal soil surface temperature forcing from equation (2). The $C_1(\delta)$ function used was equation (11) [*Hu and Isram*, 1995]. A comparison of results from the analytical solution to the HCE and the extended FRM (equation (16)) under given boundary conditions ($\overline{T}$ = 30°C, ΔT_0 = 30°C, D_a = 2 m) and where z_1 and δ are 0.05 m are shown in Figure 3. The extended FRM result coincides extremely closely to the solution of the HCE. Note that, if z_1 and δ are equal, the differences between the analytical solution to the HCE and the extended FRM calculation (equation (16)) are minimized. If z_1 and δ are of different values, then the differences between the analytical solution to the HCE and the extended FRM calculation (equation (16)) becomes larger. Figure 4 shows the relationship between distance z_1 and the maximum difference between the analytical solution to the HCE and the extended FRM under the given boundary condition. In this case, δ is taken to have the same value as z_1. The relationship provides useful information for setting z_1 appropriately. For instance, if the required accuracy is within 0.2°C, then z_1 must be less than 0.5 m. These comparisons (Figures 3 and 4) suggest that with consistent boundary conditions of surface or soil temperature, then the extended FRM can accurately calculate deeper soil temperatures.

3.2. Combined Method for Estimating Diurnal Variation in Soil Temperature

[16] Soil temperatures below depths of approximately 0.3–0.5 m can be treated as daily constants for calculating diurnal variations of soil temperature by the HCE. Diurnal changes in soil temperature below 0.3–0.5 m depth need not be considered, permitting this layer to form a lower boundary condition for the HCE. Equation (16) provides a method to calculate this lower boundary condition for diurnal soil temperature calculations. Application of equation (16) below depths of 0.3–0.5 m does not require initial temperature values or soil thermometric parameters of deep soil layers. Figure 5 shows the procedure for estimating diurnal soil temperature variations, by using the combined methods.

[17] A comparison of ground surface and soil temperature calculations between calculations using a 20 layer HCE with

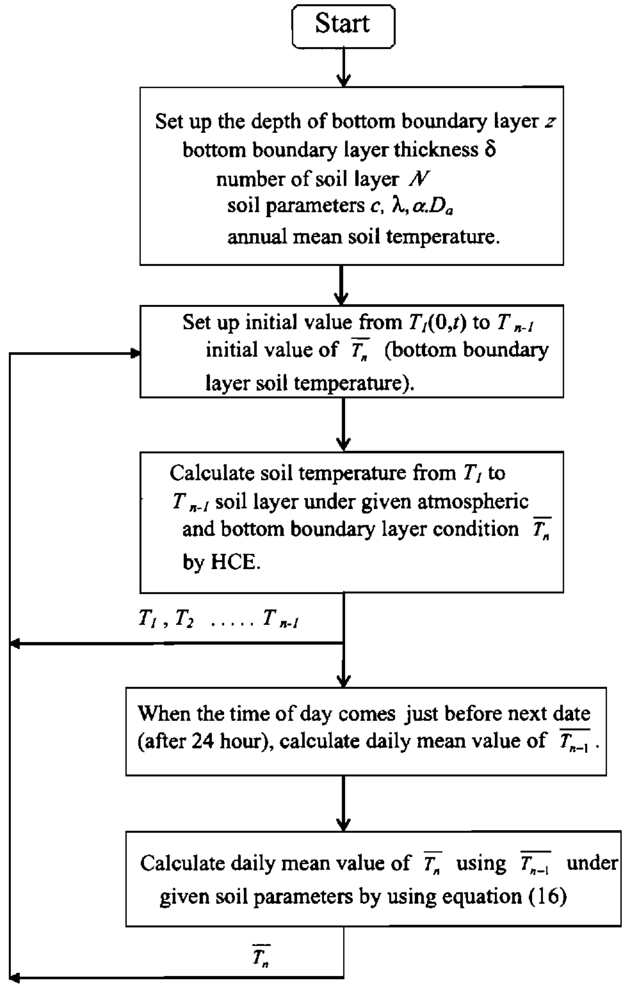

Figure 5. Schematic diagram of the procedure for estimating diurnal variations in soil temperature by combining the HCE and the extended FRM.

set boundary conditions at 10 m and a 6 layer HCE using boundary conditions at 0.5 m from equation (16) is shown in Figure 6. The calculation parameters are shown in Table 2. Simulations were calculated over a 1 year period. The differences are within 0.07°C, 0.12°C, and 0.8°C for 0 (ground surface), 0.05, and 0.5 m deep soils, respectively. The HCE modified by equation (16) also provides accurate results with a 0.3 m depth boundary condition and only a three-soil layer model. Maximum temperature differences are within 0.9°C at the 0 m (ground surface) and within 1.2°C at 0.3 m.

3.3. Combined Method for Estimating Ground Surface Temperature by Using Both the Original and Extended FRM

[18] To calculate diurnal variations in the ground surface temperature $T(0, t)$ using the original FRM (equation (12)), $\overline{T}$ is needed. Originally, $\overline{T}$ is the mean daily ground surface temperature $\overline{T(0,t)}$. *Dickinson* [1988] proposed a method of parameterization for $\overline{T}$ by coupling FRM for diurnal and annual cycles. Other methods treat $\overline{T}$ as a deep reference soil temperature [e.g., *Deardorff*, 1978; *Noihan and Planton*, 1989]. *Mihailovic et al.* [1999] examined how important the

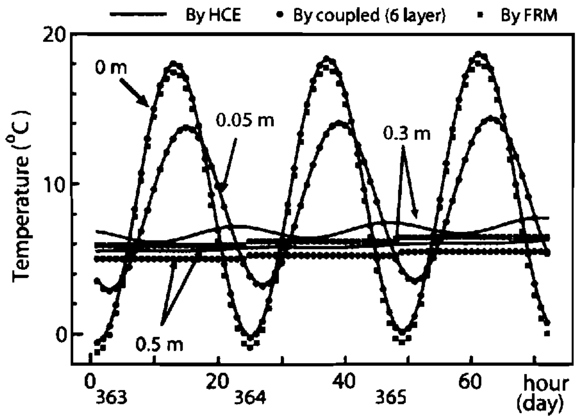

Figure 6. Comparison of diurnal variations in ground surface and soil temperature calculations using a 20-layer HCE with set boundary conditions at 10 m {HCE}, a six-layer HCE using boundary conditions at 0.5 m from equation (16) {coupled}, and the extended FRM using equations (12) and (16) with z = 0.3 m {FRM}.

deep soil temperature $\overline{T}$ is to the force-restore calculation and they proposed a new procedure for calculating the deep soil temperature based, on climatological data of soil temperature and its exponential attenuation. Although the mean daily ground surface temperature $\overline{T(0,t)}$ and mean daily soil temperature $\overline{T(z,t)}$ differ, by assuming that $\overline{T}$ is a deep soil temperature and that $\overline{T} \approx \overline{T(z,t)}$, then $\overline{T}$ can be estimated using the extended FRM (equation (16)). Figure 6 also shows example results from the HCE 20 layer calculation and the extended FRM using equations (12) and (16) with z = 0.3 m (direct FRM). The maximum errors are 1.2°C for ground surface temperatures, and 1.8°C for the $\overline{T_m}\left\{\overline{T(\delta,t)}\right\}$ at 0.3 m depth. These errors are larger than those by the extended FRM combined with the HCE.

3.4. Ground Surface Temperature Calculation Under Nonsinusoidal Forcing Condition

[19] The FRM and the extended FRM were originally derived from the assumption of sinusoidal forcing. A nonsinusoidal forcing comparison test was therefore conducted. Figure 7 compares for ground surface and soil temperatures the differences between the numerical solution of the HCE 20 layer and that using the FRM calculation under nonsinusoidal forcing. In this case, after a 366 day simulation (see sections 3.2 and 3.3), the upper boundary condition of the air temperature is constant at T_a = 10°C. This also means that the upper boundary condition provides for rapid temperature changes of about 10°C at the beginning of the calculation period for this simulation. The results of the coupled six-layer model still coincide closely to the solution of the HCE, within 0.07°C at ground surface temperature and 0.1°C at 0.1 m depth. The result of soil temperature at 0.1 m depth in the coupled three-layer model also coincide reasonably closely to the HCE, within 0.5°C. On the other hand, there are larger errors (1.0–1.8°C) in estimates of the ground surface temperature from the three-layer coupled method and the direct FRM using equations (12) and (16) with 0.3 m, particularly in the first 1–3 hours. The adjustment time of coupled three-layer model is earlier than that of the direct FRM.

[20] *Deadorff* [1978], *Dickinson* [1988], and *Hu and Isram* [1995] investigated the applicability of FRM under high-frequency nonsinusoidal conditions in detail. They concluded that although the direct FRM approach treats higher frequencies poorly, this misrepresentation of higher harmonies should usually be acceptable for purposes of estimating ground surface temperatures. The results in this paper suggest however, that the coupled HCE and FRM is a more robust method than the direct FRM for conditions of nonsinusoidal forcing.

4. Estimating Soil Temperature Under Snow Covers

4.1. Extension of Theory to Snow

[21] Snow is an excellent thermal insulator [*Sturm et al.*, 1997] and under nonmelting snow covers, the ground surface temperature is controlled more by soil temperatures than by atmospheric conditions, because the thermal conductivity of soils is generally greater than that for snow. *Pomeroy and Brun* [2001] show that subnival temperatures even in cold boreal forests are completely unrelated to daily

Table 2. Calculation Condition and Used Parameter for Figures 6 and 7

Atmospheric conditions	
No radiation and latent heat	
Only consider sensible heat $H = K(T(0,t) - Ta)$	
$Ta = Td + 15\sin(2\pi/365 \times \text{Day})$	
$Td = 10 + 20\sin(0.261799(\text{Time} - 6))$	
K = 20, *Ta* is the air temperature, Day is the number of days, and Time is the time of day	
Soil conditions	**Value**
Soil heat capacity (MJ m^{-3} K^{-1})	2.3
Soil thermal conductivity (W m^{-1} K^{-1})	1.0
Soil layer configuration (m)	
HCE 20 layers:	
Z(1) = 0.001:Z(2) = 0.01:Z(3) = 0.05:Z(4) = 0.1:Z(5) = 0.3:Z(6) = 0.5:Z(7) = 0.7:Z(8) = 1.0:	
Z(9) = 1.5:Z(10) = 2.0:Z(11) = 2.5:Z(12) = 3.0:Z(13) = 3.5:Z(14) = 4.0:Z(15) = 5.0:Z(16) = 6.0:	
Z(17) = 7.0:Z(18) = 8.0:Z(19) = 9.0:Z(20) = 10.0	
Combination of HCE and the proposed formula of six layers:	
Z(1) = 0.001:Z(2) = 0.01:Z(3) = 0.05:Z(4) = 0.1:Z(5) = 0.3:Z(6) = 0.5	
Combination of HCE and the proposed formula of three layers:	
Z(1) = 0.001:Z(2) = 0.1:Z(3) = 0.3	

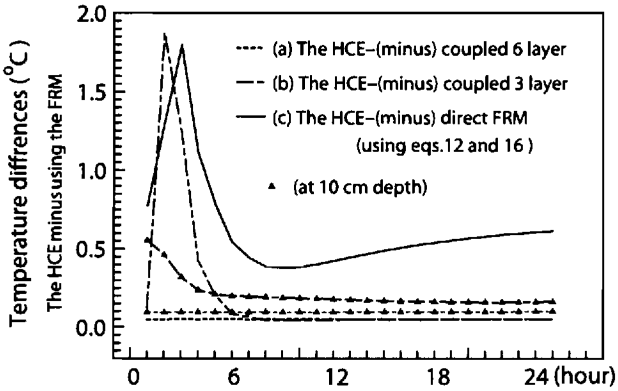

Figure 7. Comparison of variation of the temperature differences in ground surface temperature and soil temperature between the numerical solution of the HCE 20 layers and the FRM under nonsinusoidal forcing (air temperature treated as constant value (T_a = 10°C) over a day after a 366 day simulation). (a) The HCE minus coupled 6 layer using boundary conditions at 0.5 m from equation (16) {dot line; ground surface temperature, dot line with triangle mark; soil temperature at 0.1 m}. (b) The HCE minus coupled 3 layer using boundary conditions at 0.3 m from equation (16) (dot dash line; ground surface temperature, dot dash line with triangle mark; soil temperature at 0.1 m). (c) The HCE minus direct FRM (using equations (12) and (16) with z = 0.3 m; solid line).

air temperature fluctuations when snow depth exceeds a few tens of centimeters. However, the FRM presumes significant heat exchange between soil and atmosphere by assuming a periodic temperature forcing at the surface. This assumption clearly is not valid for soils under snow covers.

[22] Accepting these difficulties, application of the following models to soil under snow is attempted:

1. HCE model with the boundary condition given as the temperature observed at 0.8 m depth,
2. The original FRM (equation (10)) for 0.025 m depth.
3. The extended FRM coupled to HCE to determine the lower boundary condition 0.4 m depth.

All models were run to calculate mean daily soil temperature by using mean daily meteorological values. As mean daily winter energy inputs at the surface are comparatively small, it was assumed for the purposes of these calculations that the effect of mean daily net radiation, and latent heat flux at a ground surface under snow were negligible.

[23] The soil heat flux at the surface $G(0, t)$ is given by following equation for non-snow-covered conditions:

$$G(0,t) = -H = -K(T_s - T_a) \tag{17}$$

where H is the sensible heat flux, K is an exchange parameter for sensible heat flux between soil surface and air, T_s is the soil surface temperature, and T_a is the air temperature.

[24] A shallow snow cover can be treated as a thin soil layer for estimating mean daily temperature values, as discussed previously regarding the damping depth. Hence, heat conduction for snow cover can be approximated as

$$G(0,t) = -S = -M(T_s - T_{ss}) = -M(T_s - T_a) \tag{18}$$

where S is the heat conduction of snow between ground surface, snow, and air and T_{ss} is the snow surface temperature. M is the snow conductance parameter, given by

$$M = \lambda s / z_s \tag{19}$$

where λs is the snow thermal conductivity and z_s is the snow depth.

[25] The effect of phase change on soil temperatures was considered by replacing the latent heat of fusion with the apparent heat capacity, as outlined by *Lunardini* [1981], *Kinoshita* [1982], *Williams and Smith* [1989], *Bonan* [1991], and *Yamasaki et al.* [1998]. The volumetric latent heat of fusion is added to the volumetric heat capacity of soil over a temperature range during phase change. In this study, the temperature range used was from −1°C to 0°C. Outside of this range either the frozen or unfrozen volumetric heat capacity of soil is selected. During phase change in such conditions, the soil temperature change is small and the soil heat capacity is large compared to nonphase change conditions.

[26] The thermal conductivity of frozen and unfrozen soil was estimated following *Johansen* [1975] using autumn soil moisture values observed by Archibold et al. (Effects of burning on fescue prairie microclimate, submitted to *Canadian Field Naturalist*, 2002, hereinafter referred to as

Table 3. Parameter Used for Kernen Farm

Parameters	Values
Conductance parameter for sensible heat	0.3
Snow thermal conductivity for new snow (W m^{-1} K^{-1})	0.05
Snow thermal conductivity for melting season (W m^{-1} K^{-1})	0.2
Snow thermal conductivity except above period (W m^{-1} K^{-1})	0.08
Soil heat capacity for unfrozen soil (MJ m^{-3} K^{-1})	1.71
Soil heat capacity for frozen soil (MJ m^{-3} K^{-1})	1.37
Soil thermal conductivity for unfrozen soil (W m^{-1} K^{-1})	0.40
Soil thermal conductivity for frozen soil (W m^{-1} K^{-1})	0.56
Annual mean soil temperature (°C)	5
Soil layer configuration (m)	
HCE:	
Z(1) = 0.001:Z(2) = 0.025:Z(3) = 0.05:Z(4) = 0.1:Z(5) = 0.2:Z(6) = 0.3:Z(7) = 0.4:Z(8) = 0.8	
Combination of HCE and the proposed formula:	
Z(1) = 0.001:Z(2) = 0.025:Z(3) = 0.05:Z(4) = 0.1:Z(5) = 0.2:Z(6) = 0.3:Z(7) = 0.4	

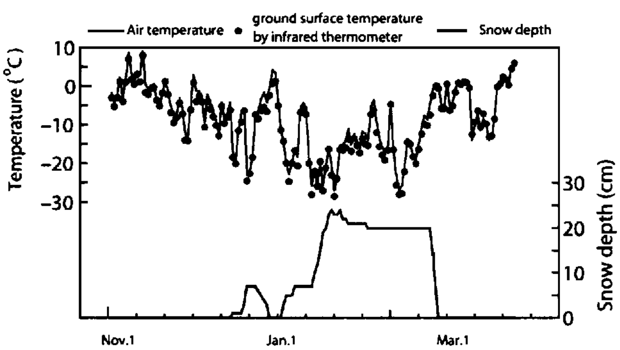

Figure 8. Temporal variations in mean daily air temperature, ground surface temperature, and snow depth, Kernen Farm, near Saskatoon, Saskatchewan, Canada.

Archibold et al., submitted manuscript, 2002). The effective thermal conductivity of snow was estimated following *Sturm et al.* [1997] using mean snow densities as estimated from techniques outlined by *Pomeroy and Gray* [1995]. Table 3 shows the parameters used for modeling.

4.2. Observations at Kernen Farm, Saskatchewan, Canada

[27] Measurements used in this study were collected from Kernen Farm, located a few kilometers east of the city of Saskatoon (52°N, 107°W) in the central southern half of the Province of Saskatchewan, Canada. This site is situated on an open level plain, covered with mixed-grass prairie. The dominant grasses are rough fescue (*Festuca altanica*), western porcupine grass (*Stipa cutriseta*) and northern wheat grass [*Ripley and Archibold*, 1999]. Soils in this area fine-textured chernozems derived from glaciolacustrine clays (Archibold et al. submitted manuscript, 2002). The climate is subhumid and typical of northern prairies with cold winters and continuous snow cover from late November to early April [*Pomeroy et al.*, 1998]. For this investigation, daily mean air temperature, daily snow depth, and daily mean soil temperature at 0.025, 0.4, 0.8, and 1.6 m depths from November to March 2000 were used. Figure 8 shows the temporal variations of daily mean air temperature, daily mean snow surface or daily mean soil surface temperature by a shielded "Everest" infrared thermometer and daily snow depth.

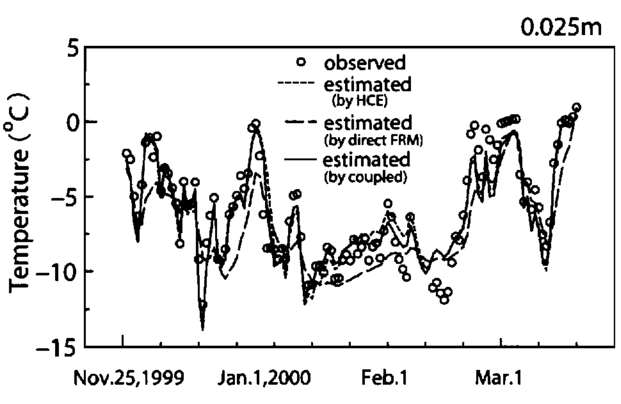

Figure 9. Estimated and measured near-surface soil temperatures (0.025 m depth) at Kernen Farm, near Saskatoon, Saskatchewan, Canada. The following methods were used for soil temperature estimation: (1) The HCE model with the boundary condition given as the temperature observed at 0.8 m depth {by HCE; dot line}, (2) The original FRM (equation (10)) for 0.025 m depth {by direct FRM; dash line}, and (3) The HCE coupled to the extended FRM (equation (16)) to determine the lower boundary condition 0.4 m depth {by coupled; solid line}.

4.3. Implementation

[28] During the observation period, mean daily snow surface temperature T_{ss} was almost equal to the mean daily air temperature T_a (Figure 8). Therefore, equation (18) was used to estimate the conductive heat flux between snow and soil surface.

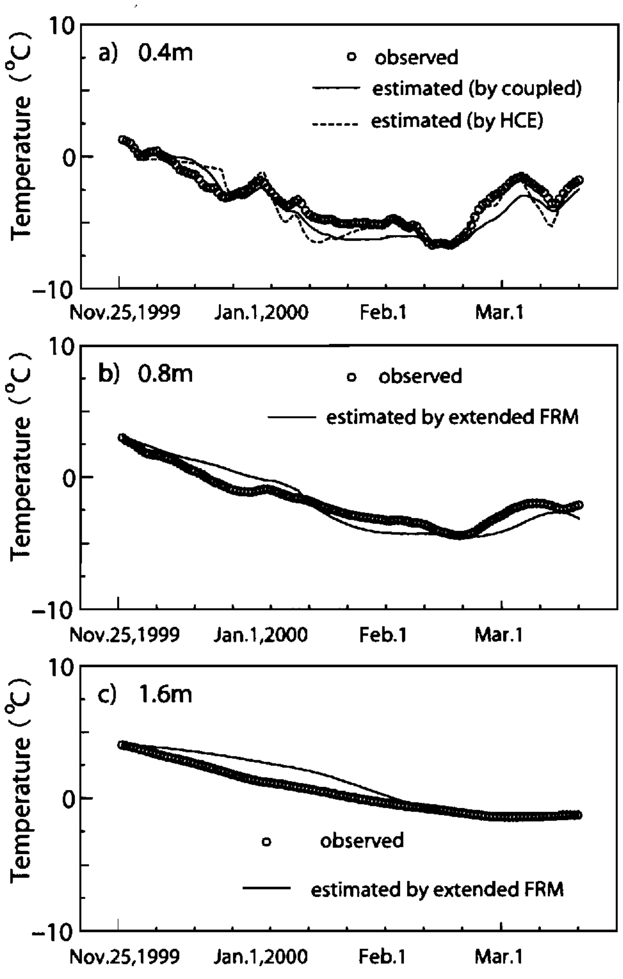

Figure 10. Estimated and measured soil temperatures at Kernen Farm, near Saskatoon, Saskatchewan, Canada. (a) Soil temperature at 0.4 m depth: estimations values obtained by coupled (solid line) and HCE (dot line), (b) soil temperature at 0.8 m depth: estimation by using the extend FRM (equation (16)) and using results of an estimation where soil temperature at 0.4 m depth is coupled as a upper boundary condition, and (c) soil temperature at 1.6 m depth: estimation result by using same as (b).

5. Results

[29] Figure 9 shows soil temperatures observed and calculated by three methods. The first method (estimated by full HCE) and third method (extended FRM coupled to reduced HCE) agree well with observed values. Root mean square errors are 1.9°C by the first method, and 1.6°C by the third. The second method (using original FRM) did not agree well with observations during the snow-covered period, with a root mean square error of 4.6°C. These results suggest that the mean daily soil temperature under snow cover in midwinter can be estimated from mean daily air temperature, snow density and snow depth, without considering net radiation and latent heat.

[30] During the snowmelt season, from the end of February to the beginning of March, all modeled values were underestimated compared to observations. The underestimation is likely due to the model implementation not considering the additional energetics of infiltrating meltwater into frozen soils [*Pomeroy et al.*, 1998]. Meltwater percolates heterogeneously through the snowpack, warming its base to 0°C [*Marsh and Woo*, 1984] and then begins to infiltrate into the frozen ground; upon refreezing in the soil it releases large quantities of latent heat, which dramatically warm the soil [*Zhao and Gray*, 1999]. Any model of soil temperature will need to consider the effect of meltwater infiltration to frozen soils to accurately estimate soil temperatures during the melt period.

[31] The lower boundary condition of the third method is at 0.4 m depth. As mentioned in sections 2.4 and 3.1, the extended FRM using equation (16) can calculate deep mean daily soil temperatures $\overline{T(z,t)}$ using the temperature of upper soil layer, under given parameters (λ, c, D_a and $\overline{T_{ym}}$); therefore soil temperatures at depths below 0.4 m can be calculated using equation (16) and the soil temperature at 0.4 m depth. The mean daily soil temperatures at 0.5, 0.6, 0.7, 0.8, 1.0, 1.2, 1.4, and 1.6 m were calculated. Figures 10b and 10c compare the observed soil temperatures at 0.8 and 1.6 m depth and values estimated by using the 0.4 m value, estimated by the extended FRM coupled to a reduced HCE (Figure 10a) and extrapolated using equation (16). Estimated soil temperatures matched observations reasonably well. Therefore, it is expected that this method can also be used to estimate deep soil temperature in frozen conditions.

6. Conclusions

[32] A simple formula to estimate seasonal variations in deep soil temperatures was developed using an extended force-restore approach. This formula can be used to accurately calculate the daily mean soil temperature at depth without considering deep soil thermometric conditions. Combining the heat conduction and extended FRM, the combined method avoids part of limitations of both routines and provides quite satisfactory representations of not only mean daily but also diurnal variations in soil temperatures. Its advantages are substantial savings in computational time and an easier parameterization than the full, multilayered HCE calculations. The force-restore approach was extended from calculation of ground surface temperature to the calculation of deep soil temperature and in frozen conditions. This was demonstrated in a cold continental climate by using observations from Saskatchewan, Canada. To apply the method accurately to a wide variety of surface and climate conditions it needs to be coupled to a surface energy balance model and to consider the effect of meltwater infiltration to frozen soils on soil temperature.

[33] **Acknowledgments.** The authors wish to acknowledge the effort and assistance of D. K. Bayne in maintaining a successful field measurement site at Kernen farm. The authors also thank N. R. Hedstorm and the National Water Research Institute, Environment Canada, for supporting observations at Kernen farm. Thanks are extended to K. Yamada and R. Sameshima of the National Agricultural Research Center for Hokkaido Region, National Agricultural Research Organization, Japan, for their appreciation and support of this research. The authors are obliged to the anonymous referees for giving valuable comments. This work was performed during the first author's stay as a visiting professor at University of Saskatchewan, Department of Agricultural and Bioresource Engineering. The stay was financially supported by the Science and Technology Agency of the Government of Japan.

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