Extension of the Force Restore Method for Estimating Deep Soil Temperature and its Application to a Cold Continental Region for Estimating Frozen Soil Depth

Tomoyoshi Hirota[1], John Pomeroy[2], Raoul Granger[3] and Charles Maule[4]
[1]*National Agricultural Research Center for Hokkaido Region, Sapporo, Japan*
[2]*Institute of Geography and Earth Sciences, University of Wales, Aberystwyth, Ceredigion, Wales, UK*
[3]*National Water Research Institute, National Hydrology Research Centre, Saskatoon, SK, Canada*
[4]*Dept of Agriculture and Bioresource Engineering, University of Saskatchewan, Saskatoon SK, Canada*

Abstract

The force restore method was originally developed for estimating diurnal fluctuations in ground surface temperature. Because of its relatively simple parameterization, it is commonly applied in meteorological and other models for this purpose. Its application to the calculation of deeper soil temperatures, frozen soils and soils under snow cover has heretofore not been possible. This study demonstrates an extension of the force-restore method that permits accurate estimates of seasonal variation in deep soil temperature; frozen soil depth and ground surface temperature under a snow cover. The extended method is tested using measurements collected in a cold continental region. The modified formula can also be applied to determine the lower boundary condition for calculations of diurnal variations in soil temperature.

1. Introduction

The force restore method (FRM) is an alternative approach, developed to estimate the ground surface temperature. Hu and Isram (1995) showed that the FRM could provide accurate estimates of both ground surface and upper soil temperatures by minimizing the error between the analytical solution from the force-restore method and from that of the heat conduction equation under diurnal forcing. Hirota *et al.*, (1995) demonstrated and tested with field measurements in Japan, an extension of the FRM to estimate the seasonal variation in daily mean soil temperature of shallow (upper) soil layers. However, these versions of the FRM did not consider estimating deep soil temperature, nor have they been fully tested under snow cover and for frozen soils. It is doubtful that existing FRM formulations can accurately represent the ground surface temperature under a snow cover as they assume a strong diurnal 'forcing' at the surface. Insulation of soil from air by snow cover will strongly dampen diurnal temperature fluctuations at the ground surface and may violate the force-restore assumption. The prospects of successfully applying an unmodified FRM approach to cold, snow covered regions are therefore questionable.

This study proposes a new and simple method for estimating deep soil temperature using a modification of the FRM. The FRM can predict diurnal variations in ground surface temperature if appropriate boundary conditions can be specified. Here, extended methods for seasonal variations in mean daily deep soil temperature are shown. The new method also can be applied to determine lower boundary condition of soil temperature to estimate ground surface temperature or vertical soil temperature profile. It can be applied for not only mean daily value but also diurnal variations. This application of the FRM is also effective for estimating daily boundary conditions and estimating soil temperature profile for frozen and unfrozen soil under snow. The result of the extended FRM is compared to measurements of soil temperature regimes over a winter near Saskatoon, Saskatchewan, Canada.

2. Extension of the Force Restore Method

2.1 Review of Method

The sinusoidal soil temperature changes is expressed by

$$T(z,t) = \overline{T} + \Delta T_0 e^{-z/D_a} \sin\left(\omega t - z/D_a\right) \tag{1}$$

where $T(z,t)$ is the soil temperature over some vertical coordinate, z, and time t, $\overline{T}$ is the mean ground surface temperature (daily or annual), ΔT_0 is the daily or annual temperature amplitude at the surface, and ω is the frequency of oscillation equal to $2\pi/\tau$, $D_a = (2\alpha/\omega)^{0.5}$, D_a is the damping depth (m) of surface temperature fluctuations, τ is the period of temperature fluctuation calculation (day or year), α is the soil thermal diffusivity, found as $\alpha = \lambda/c$, λ is the soil thermal conductivity, c is the volumetric heat capacity.

The vertical conductive heat flux in a soil, G, at depth z and time t is given by

$$G(z,t) = -\lambda \frac{\partial T(z,t)}{\partial z} \tag{2}$$

Combining Eqs. 2 and 1 provides

$$G(z,t) = \left(\frac{\omega c \lambda}{2}\right)^{0.5} \left(\omega^{-1} \frac{\partial T(z,t)}{\partial t} + T(z,t) - \overline{T}\right) \tag{3}$$

which is a differential form of the soil heat flux with respect to time (t). Eq. 3 provides the soil heat flux differentiated with respect to distance and time (Bhumralkar, 1975).

2.2 Application to the Soil Surface Layer

Considering a soil surface layer of thickness below the ground surface, as shown in Figure 1, then the rate of temperature change over time for this layer is given by

$$c \frac{\partial T_g(\delta,t)}{\partial t} = -\left(\frac{G(\delta,t) - G(0,t)}{\delta}\right) \tag{4}$$

T_g, the ground surface temperature is defined as

$$T_g(\delta,t) = \frac{1}{\delta}\int_0^{\delta} T(z,t) \qquad (5)$$

Assuming that T (δ,t) $\approx$ T_g (δ,t), and combining Eqs. 5 and 4, we obtain

$$C_1 \frac{\partial T_g(\delta,t)}{\partial t} = \frac{2}{cD_a} G(0,t) - \frac{2\pi}{\tau}(T_g(\delta,t) - \overline{T}) \qquad (6)$$

Here, C_1 is the function of δ and D_a. Hu and Isram (1995) developed C_1 function to minimize the difference between the analytical solution of the FRM and the full heat conduction equation (HCE) in response to a single periodic forcing. Their polynomial approximation is

$$C_1 = 1 + 0.943(\delta / D_a) + 0.223(\delta / D_a)^2 + 1.68 \times 10^{-2}(\delta / D_a)^3 - 5.27\ 10^{-3}(\delta / D_a)^4 \qquad (7)$$

For case of $\delta \to 0$ in Eq. 7,

$$\frac{\partial T(0,t)}{\partial t} = \frac{2}{cD_a} G(0,t) - \frac{2\pi}{\tau}(T(0,t) - \overline{T}) \qquad (8)$$

This, in the same form as the original, is a FRM of ground surface temperature for which T(0,t) = T_g(0,t).

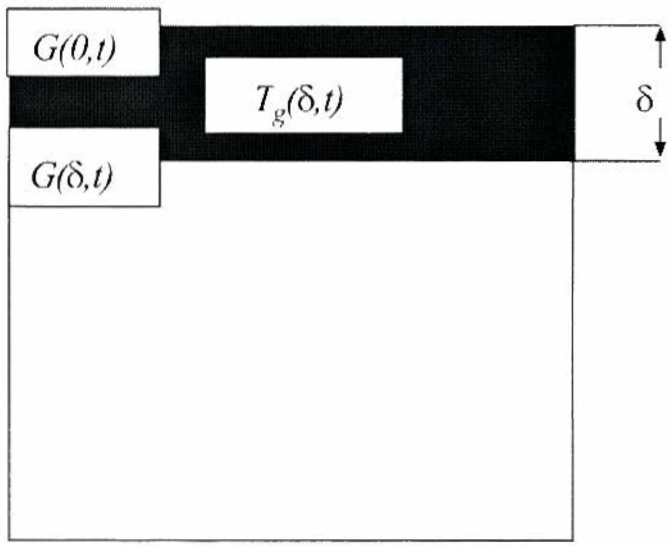

Figure 1: Schematic diagram of the Force-Restore for a soil surface.

2.3 Extension to Mean Daily Soil Temperature

The FRM can be applied from Eq. 8 to estimate variations in ground surface temperature, however its extension to calculation of soil temperature has been restricted for several reasons. It has been maintained that determining the daily mean ground surface temperature is problematic in that a value of $\overline{T}$ is required before solving for the diurnal variations of soil temperature using the FRM (e.g., Mihailovic *et al.*, 1999). In addition, $\overline{T}$ is required at depth to provide a lower boundary condition for diurnal calculations. The value of $\overline{T}$ may also have to respond to changing surface thickness, δ from day to day.

Application to annual variations may be less of a problem as field measurements show that the mean annual soil temperature, $\overline{T_{ym}}$, is relatively invariant with depth. Therefore $\overline{T_{ym}}$ can be treated as a constant for a location, and its value need not be changed with changing δ. $\overline{T_{ym}}$ at a location can be estimated using well-tested, simple empirical equations from the mean annual air temperature, permitting a relatively easy parameterization of the FRM for calculations of annual variation in mean daily soil temperature (Hirota *et al.*, 1995).

2.4 Extension to Mean Daily Soil Temperatures at Depth

When applying Eq. 4 to an internal soil layer to estimate daily mean soil temperature as shown in Figure 2, then the time rate of temperature change for this layer is given by

$$c\frac{\partial \overline{T(z,t)}}{\partial t} = -\left(\frac{G_n - G_{n-1}}{\delta}\right) \tag{9}$$

Combining Eqs. 3 and 9 provide

$$C1\frac{\partial \overline{T(z,t)}}{\partial t} = \frac{2}{cD_a}G_{n-1} - \frac{2\pi}{\tau_y}(\overline{T(z,t)} - \overline{T_{ym}}) \tag{10}$$

Here, $\overline{T(z,t)}$ is the daily mean soil temperature, τ_y is the annual period (365 days) and G_{n-1} is the daily mean soil heat flux between an upper and internal soil layer; expressed as follows

$$G_{n-1} = -\lambda\frac{\partial \overline{T(z,t)}}{\partial z_1} \tag{11}$$

Combining Eqs. 10 and 11 obtains

$$C_1(\delta)\frac{\partial \overline{T(z,t)}}{\partial t} = -\frac{2\lambda}{cD_a}\frac{\partial \overline{T(z,t)}}{\partial z_1} - \frac{2\pi}{\tau}(\overline{T(z,t)} - \overline{T_{ym}}) \tag{12}$$

where z_1 is the distance from upper soil depth to internal soil depth.

Note that when solving the heat conduction equation (HCE), $\partial T/\partial t = \lambda/c \cdot \partial^2 T/\partial z^2$, for annual variations in soil temperature, it is necessary to set a lower boundary condition at several to several tens of metres depth. At the lower boundary, soil temperature is constant or the soil heat flux is zero. However, the extended FRM using Eq. 12 does not need to make such assumptions about deep soil conditions, permitting flexible lower boundary conditions that can be provided to a calculation such as the HCE.

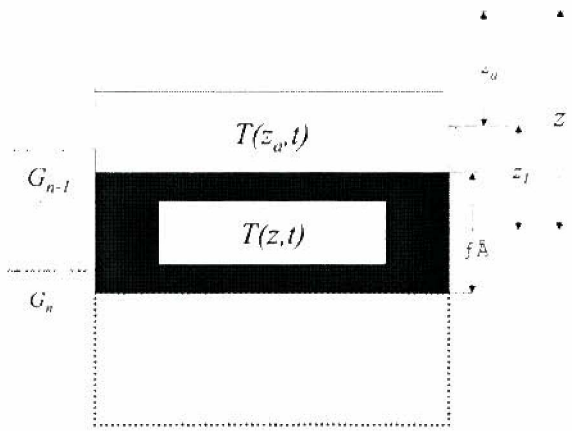

Figure 2: Schematic diagram for estimating deeper soil temperature by using the Force-Restore approach.

3. Application of the Extended FRM

3.1 Comparison to the Heat Conduction Equation Given Set Boundary Condition

The extended FRM for deep soil temperature calculation was compared to an analytical solution to the heat conduction equation {HCE (Eq. 1)}. This comparison used a sinusoidal soil surface temperature forcing from Eq. 1 when z=0. A comparison of results from the analytical solution to the HCE and the extended FRM (Eq. 12) under given boundary conditions ($\overline{T}$ = 10°C, ΔT_0 = 30°C, D_a = 2 m) is shown in Figure 3. The extended FRM result coincides extremely closely to the solution of the HCE.

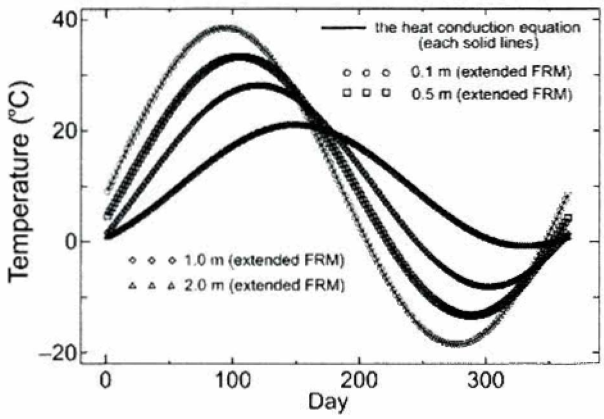

Figure 3: Comparison of soil temperature between the analytical solution of the heat conduction equation and the extended Force-Restore Method ($z_1 = 0.05$ m).

3.2 Combined Method for Estimating Diurnal Variation in Soil Temperature

Soil temperatures below depths of approximately 0.3 to 0.5 m can be treated as daily constants for calculating diurnal variations of soil temperature by the HCE. Diurnal changes in soil temperature below 0.3-0.5 m depth need not be considered, permitting this layer to form a lower boundary condition for the HCE. Eq. 12 provides a method to calculate this lower boundary condition for diurnal soil temperature calculations (Figure 4). Application of Eq. 12 below depths of 0.3-0.5 m does not require initial temperature values or soil thermometric parameters of deep soil layers. A comparison of ground surface and soil temperature calculations between calculations using a 20 layer HCE set boundary conditions at 10 m, and a 6 layer HCE using boundary conditions at 0.5 m from Eq. 12 is shown in Figure 5. Simulation was carried out one-year period. The differences are within 0.07 °C and 0.8 °C for ground surface and 0.5 m deep soil respectively.

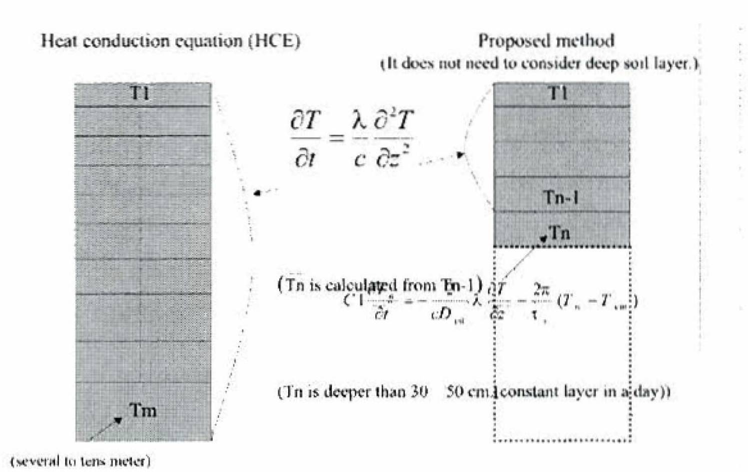

Figure 4: Concept of combination of heat conduction equation and the proposed method.

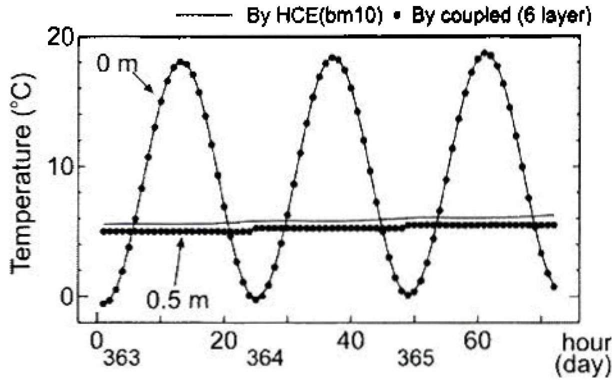

Figure 5: Comparison of diurnal variations in soil temperature.

4. Estimating Soil Temperature under Snow Covers

Snow is an excellent thermal insulator and under non-melting snow covers, the soil surface temperature is controlled more by soil temperatures than by atmospheric conditions, because the thermal conductivity of soils is generally greater than that for snow. However, the FRM presumes significant heat exchange between soil and atmosphere by assuming a periodic temperature forcing at the surface. This assumption clearly is not valid for soils under snow covers.

Accepting these difficulties, application of the following models to soil under snow is attempted:

1. HCE model with the boundary condition given as the temperature observed at 0.8 m depth,
2. the original FRM (Eq. 6) for 0.025 m depth, and
3. the extended FRM coupled to HCE to determine the lower boundary condition 0.4 m depth.

All models were run to calculate mean daily soil temperature by using mean daily meteorological values. As mean daily winter energy inputs at the surface are comparatively small, it was assumed for the purposes of these calculations that the effect of mean daily net radiation, and latent heat flux at a ground surface under snow were negligible.

4.1 Results

Measurements used in this study were collected from Kernen Farm, located a few kilometres east of the city of Saskatoon (52°N, 107°W) in the central southern half of the Province of Saskatchewan, Canada. Figure 6 shows soil temperatures observed and calculated by three methods. The first method (estimated by full HCE) and third method (extended FRM coupled to reduced HCE) agree well with observed values. Root mean square errors are 1.9°C by the first method, and 1.6°C by the third. The second method (using original FRM) did not agree well with observations during the snow-covered period, with a root mean square error of 4.6°C. These results suggest that the mean daily soil temperature under snow cover in mid-winter can be estimated from mean daily air temperature, snow density and snow depth, without considering net radiation and latent heat.

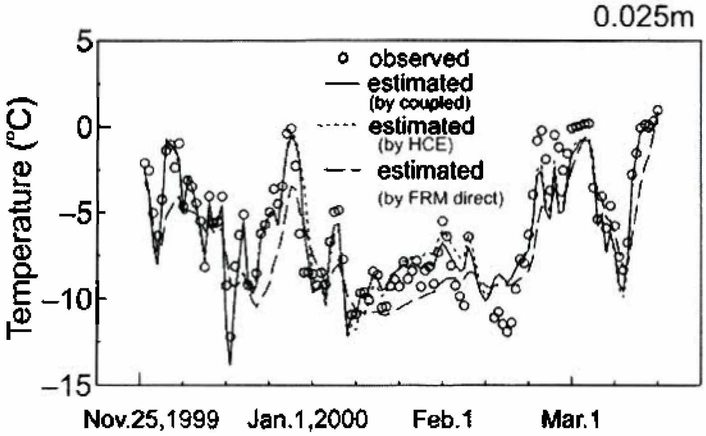

Figure 6: Estimated and measured near-surface soil termperatures at Kernen farm.

During the snowmelt season, from the end of February to the beginning of March, all modelled values were underestimated compared to observations. The underestimation is likely due to the model implementation not considering the additional energetics of infiltrating meltwater into frozen soils (Pomeroy *et al.*, 1998). Any model of soil temperature will need to consider the effect of meltwater infiltration to frozen soils to accurately estimate soil temperatures during the melt period.

The boundary condition of the third method is at 0.4 m depth. To estimate soil temperatures at depths below this, Eq. 12 can be used to extrapolate downward. Figure 7 shows the resulting comparison of observed soil temperatures at 0.8 m and 1.6 m depth and estimated values by using the 0.4 m estimated value by the third method and extrapolating using Eq. 12. Estimated soil temperatures matched observations reasonably well. Therefore this method can also be used to estimate deep soil temperatures in frozen conditions.

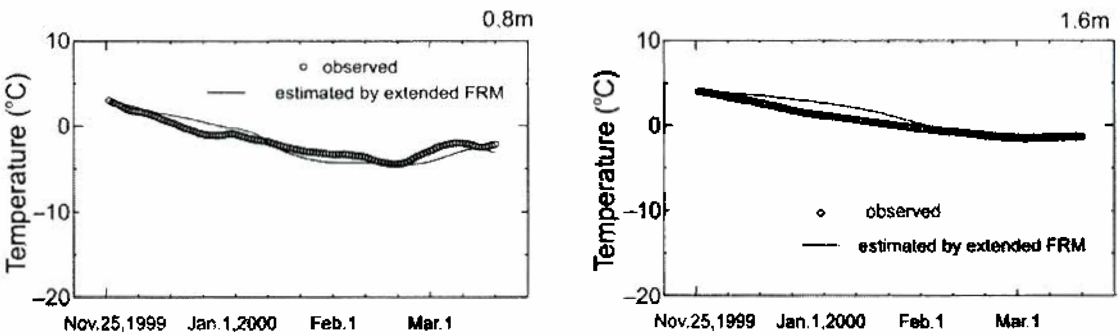

Figure 7: Estimated and measured deep soil temperatures at Kernen farm.

5. Conclusions

A simple formula to estimate seasonal variations in deep soil temperatures was developed using the force-restore approach. This formula can be used to accurately calculate the daily mean soil temperature at depth without considering deep soil thermometric conditions. Its advantages are a substantial savings in computational capacity and easier parameterization than full, multi-layered heat capacity equation calculations. The force-restore approach was extended from calculation of ground surface temperature to the calculation of deep soil temperature and frozen soil depth. This was demonstrated in a cold continental climate by using observations from Saskatchewan, Canada. To apply the method accurately to a wide variety of surface and climate conditions it needs to be coupled to a surface energy balance model and to consider the effect of meltwater infiltration to frozen soils on soil temperature.

6. References

Bhumralkar, C.M., 1975: Numerical experiment on the computation of ground surface temperature in an atmospheric circulation model. *J. Appl. Meteor.*, **14**: 1246-1258.

Hirota, T., Fukumoto, M., Shirooka ,R. and K. Muramatsu, 1995: Simple method of estimating daily mean soil temperature by using Force-Restore Model. *J. Agr. Meteorol.*, **51**: 269-277. (in Japanese with English abstract)

Hu, Z and S. Isram, 1995: Prediction of ground surface temperature and soil moisture content by the force-restore method. *Water Resour. Res.*, **31**: 2531-2539.

Mihailovic, D.T., Kallos, G., Arsenic, I.D., Lailic, B., Rajkovic, B. and A. Papadopoulos, 1999: Sensitivity of soil surface temperature in a Force-Restore Equation to heat fluxes and deep soil temperature. *Int. J. Climatol*, **19**: 1617-1632.

Pomeroy, J.W., Gray, D.M., Shook, K.R., Toth, B., Essery, R.J.H., Pietroniro, A. and N. Herdstorm, 1998: An evaluation of snow accumulation and ablation processes for land surface modeling. *Hydrol. Process.*, **12**: 2339-2367.