PHYSICAL MODELLING OF BLOWING SNOW FOR AGRICULTURAL PRODUCTION

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ABSTRACT

This study attempts to estimate the quantities of surface snow transported and sublimated during individual blowing snow events and for longer time periods. The model is based on physical meteorological relationships which describe blowing snow transport and sublimation. It uses standard meteorological, land use and biophysical terrain data. Application of the model to estimate blowing snow transport and sublimation over various land uses and climatic zones of the Canadian Prairies is made. These data can determine the effects of various agricultural practices and vegetative covers on snow transport and sublimation given specific meteorlogical patterns. Such information is useful in assessing the feasibility of snow management to enhance the water supplies available for spring soil moisture recharge across the northern plains.

In Proceedings, Snow Management for Agriculture Symposium. H. Steppuhn ed. Lincoln, NB: Great Plains Agricultural Council Publication ___. In press.

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INTRODUCTION

Agricultural snow management involves modifying the land surface roughness, thereby decreasing the ability of the wind to entrain and transport snow. Given a particular snow management practice, it is of interest to know how much snow is available for trapping as a result of the local climate, land use and terrain. A physically based blowing snow model, using input from weather stations, aerial photographs and topographic maps will help identify the availability of blowing snow at a given location and help predict the effects of various management practices for trapping snow. Tabler (1975) demonstrated a blowing snow model based on accumulation patterns and noted that the individual processes were not sufficiently understood to allow physically based modelling. Rapid development in the understanding of multi-phase flows in the last decade as evidenced in reviews by Male (1980, 1984) and Schmidt (1982a) suggest that the development of a physically based model is now possible.

DEVELOPMENT OF THE MODEL

The model described below is an initial attempt to estimate the quantities of surface snow transported and sublimated during individual blowing snow events and for longer time periods. It is based on our current understanding of snow transport processes as published in the literature and on the results of measurements made at Loreburn, Saskatchewan over the winter of 1984-85. The model first describes "fully developed" blowing snow, where horizontal transport levels are at a maximum for a given set of parameters describing atmospheric turbulence and density, snowcover supply and snowpack cohesion. The model is then adapted to various terrain conditions where the flow is not fully developed.

Fully developed blowing snow may be modelled in terms of the control volume illustrated in Fig. 1. The components of mass transfer across the boundaries are the horizontal flow of snow particles into and out of the volume, the vertical flux of snow particles across the top control surface, the erosion/deposition flux of snow at the snow surface, the horizontal inflow and outflow of water vapour and the vertical flux of water vapour through the top surface. For fully developed blowing snow, the horizontal inflow is equal to the horizontal outflow in the mean and it is assumed that the horizontal vapour flux is constant in the mean. The height of the control volume is set at 10 m since most blowing snow transport occurs below that level (Kobayashi, 1972). Also the vast majority of snow particles at 10 m are unable to return to the surface before sublimating.

Horizontal Snow Transport

The horizontal transport of snow occurs in two modes, saltation and suspension. Saltation involves the bouncing of particles in a curved trajectory near the surface. When a particle strikes the surface, momentum is transferred to surface particles which in turn are ejected from the snowpack. Saltation is the mechanism by which particles are removed from the snowpack and made available for suspension. Suspension occurs when lift forces on the particle due to vertical atmospheric turbulence are greater than gravitational forces.

Saltation

An expression developed by Iversen et al (1975) has been demonstrated by Schmidt (1982a) to predict the saltating flux of blowing snow as a function of the mean saltating particle fall velocity $\overline{\omega}$, the aerodynamic friction velocity u^* and the friction velocity at the threshold of saltation u^*_t (threshold friction velocity). This expression has the form

$$Q_{salt} = C(\rho/g)(\overline{\omega}/u^*_t)\ u^{*2}(u^*-u^*_t)\ . \qquad (1)$$

Based on saltation data from D. Kobayashi (1972) and S. Kobayashi (1978), Schmidt concludes that the proportionality coefficient $C \simeq 1$ for blowing snow. The gravitational constant $g = 9.8\ m/s^2$ and the atmospheric density, ρ, is the sum of the dry air and water vapour densities. Results from the Loreburn experiments indicate that Tabler's (1980) relationship between z_o the roughness height, u^* and the 10-m windspeed, u_{10}, over lake snow is valid for flat to slightly undulating summerfallow fields and snow-filled stubble fields with complete snowcover. Thus,

$$u^* = 0.024\ u_{10}^{1.18}\ , \qquad (2)$$

and $$z_o = 1.35 \cdot 10^{-3} u*^2 \quad , \tag{3}$$

where z_o is in metres and u_{10} and $u*$ are in m/s.

When the snow is not blowing or is at the threshold of saltation

$$z_o = 1.6 \cdot 10^{-5} (m) \quad . \tag{4}$$

Assuming a neutral atmosphere, the friction velocity during non-blowing or threshold conditions is found using the logarithmic wind profile relationship,

$$u* = \frac{u_{10} \cdot k}{\ln(10/z_o)} = 0.02997\ u_{10} \quad . \tag{5}$$

Equation 5 applies to a fallow or completely snowfilled stubble field. k is the von Kármán constant (0.4).

For saltating snow over an exposed stubble field both z_o and $u*$ are increased by the shear of the stubble stalks on the wind. Measurements at Loreburn over exposed wheat stubble 10 cm in height gave the following relationships between $u*$, z_o and u_{10}:

$$u* = 0.022\ u_{10}^{1.32} \tag{6}$$

and $$z_o = 0.0073\ u*^2 \quad . \tag{7}$$

For non-blowing snow conditions $$z_o = 5.0 \cdot 10^{-4} (m) \quad , \tag{8}$$

and $$u* = \frac{u_{10} \cdot k}{\ln(10/z_o)} = 0.04039\ u_{10} \quad . \tag{9}$$

The threshold friction velocity can be determined from observations of the occurrence of blowing snow and the 10-m windspeed. The Atmospheric Environment Service (AES) observes the presence of blowing snow on short grass surfaces which are generally completely filled by snow. Therefore, estimates of the threshold based on this data correspond to that for a fallow or filled-in stubble field. Data for the threshold friction velocity on exposed stubble are not available and must be determined empirically. Lyles and Allison (1976) have developed a relationship for calculating the dimensionless ratio C_1 of the threshold friction velocity for stubble to that for fallow for the case of blowing soil. The relationship uses the silhouette area and geometry of the stalks and is based on wind tunnel observations. The expression has the form

$$C_1 = 1.638 + 17.04\ N_{st} A_{st} - 0.117\ Ly/Lx \quad , \tag{10}$$

where C_1 = the ratio $u_t^*(\text{stubble})/u_t^*(\text{fallow})$,

N_{st} = the number of stubble stalks per unit area,

A_{st} = silhouette area of a single stalk,

Ly = distance between stalks parallel to the wind vector,

and Lx = distance between stalks perpendicular to the wind vector.

Since Eq. 10 accounts for only the effect of the stubble roughness elements it is assumed to be equally applicable to blowing snow.

The use of Eq. 1 for blowing snow over stubble assumes that the shear stress on the stubble stalks remains constant at its threshold level. This follows from Owen's (1964) proposal that in saltation, the surface shear stress is constant at its threshold level and the difference between the shear stress above the saltating layer and the surface shear stress is that available to "drive" saltation. However, stubble stalks are 5 to 50 cm above the saltating layer and would not experience the wind velocity modification within the saltation layer. In terms of the total shear stress, τ, over stubble

$$\tau = \tau_s + \tau_b + \tau_r \quad , \tag{11}$$

where τ_s = the shear stress exerted directly on the snow surface,

τ_b = the shear stress exerted by saltating particles on the surface,

and τ_r = the shear stress exerted on the stubble roughness elements.

If the cohesion and snow structure of stubble and fallow snowpacks can be considered similar (an untested assumption), the threshold shear stress on fallow will be equal to τ_s. In terms of friction velocities, the portion of the threshold friction velocity corresponding to the shear stress on stubble stalks is therefore

$$u_s^* - u_f^* = (1 - \frac{1}{C_1})u_s^* \ . \tag{12}$$

where u_s^* = the friction velocity for saltation in a stubble field,

u_f^* = the friction velocity for saltation in a fallow field.

In Eq. 1, $(u^* - u_t^*) = u_{salt}^*$ represents the friction velocity available to drive the saltation mechanism. The friction velocity available for saltation over stubble is

$$u_{salt}^* = \left[u_s^* - (u_{tf}^* + (1 - \frac{1}{C_1})u_s^*\right] \quad , \tag{13}$$

where u^*_{tf} = the threshold friction velocity for a pack without stubble.

Equation 1 can now be generalized for fallow or stubble fields giving

$$Q_{salt} = C(\rho/g)(\overline{\omega}/u^*_t)u^{*2}(u^* - \{u^*_{tf} + (1 - \frac{1}{C_1})u^*\}) \quad . \tag{14}$$

The average drift density in saltation, η_{salt}, is the quantity from which the lower layers of suspended transport are calculated. It is a function of the mean horizontal particle velocity, $\overline{u}_p$, and the mean height of saltation trajectories $\overline{h}$, where,

$$\eta_{salt} = \frac{Q_{salt}}{\overline{u}_p \ \overline{h}} \quad . \tag{15}$$

The dynamics of the saltation process can be examined to determine $\overline{u}_p$ and $\overline{h}$. A saltating particle has a horizontal velocity component u_p and a vertical component w_p (see Fig. 2). From a detailed analysis of particle trajectories Kawamura (1948) found that $(u_{p_1} + u_{p_2})/L \simeq g/w_{p_1}$. Hence

$$\overline{u}_p = \frac{(u_{p_1} + u_{p_2})}{2} = \frac{Lg}{2w_{p_1}} \tag{16}$$

where L = the horizontal length of the saltation trajectory,

and w_{p_1} = the initial vertical velocity component.

Based on measurements of saltation trajectories under natural conditions, Kobayashi (1972) proposed the following empirical relationship for fully developed conditions,

$$L = 0.011 \ u_1 \quad , \tag{17}$$

where u_1 = windspeed at 1 m. This allows a solution for L based on the logarithmic wind profile and the 10-m windspeed. Bagnold (1941) proposes a linear relation between w_{p_1} and u^*. Owen (1964) shows that for a solid saltating in a gas

$$w_{p_1} = 0.25u^* + \overline{\omega}/3 \quad . \tag{18}$$

Substitution of Eq. 17 and Eq. 18 into Eq. 16 gives

$$\overline{u}_p = \frac{0.011u_1 \ g}{0.5u^* + \frac{2\overline{\omega}}{3}} \quad . \tag{19}$$

$\overline{h}$ is found from the expression (Bagnold, 1941; Owen, 1964)

$$\overline{h} = \frac{w_1^2}{2g} \quad . \tag{20}$$

Combining Eqs. 15, 18, 19 and 20 gives

$$\eta_{salt} = \frac{363.6\ Q_{salt}}{u_1(0.25u^* + \overline{\omega}/3)} \quad . \tag{21}$$

u^* and u_1 are determined from the 10 m windspeed using the logarithmic profile and appropriate roughness parameters.

Suspension

In order to calculate the horizontal snow transport in suspension a reference drift density is required. This density can be determined using the probability that a particle in saltation will enter suspension. Radok (1968) suggests that if the velocity of an upward turbulent eddy exceeds a particle's fall velocity that particle will enter the suspended mode of transport. Using this idea, the two probability distributions required are those of the upward turbulent velocity and the snow particle fall velocity at the top of the saltation layer.

The probability that a vertical turbulent velocity w will exceed some value has been shown by Panofsky and McCormick (1960) to be normally distributed with a mean of zero and variance of au^*, where $a = 1.1$. Transforming w into a standard normal variate, the probability that the variate will exceed the transformed fall velocity is

$$P\left(\frac{w}{au^*} \geq \frac{\omega}{au^*}\right) = 1 - \frac{1}{\sqrt{2\omega}} \int_{-\infty}^{\frac{\omega}{au^*}} e^{-x^2/2}\, dx \quad . \tag{22}$$

The mean of this distribution is 0 with variance = 1. Considering the upward turbulence velocities only and since they occur 50% of the time, the probability that an upward velocity $w\uparrow$ will exceed some value is

$$P\left(\frac{w\uparrow}{au^*} \geq \frac{\omega}{au^*}\right) = 2 - (2/\pi)^{0.5} \int_{-\infty}^{\frac{\omega}{au^*}} e^{-x^2/2}\, dx \quad . \tag{23}$$

The distribution of particle radii, P_r, in saltation has been confirmed by Schmidt (1981) to fit the two-parameter gamma distribution:

$$f(P_r) = \frac{P_r^{(\alpha-1)}}{\beta^{\alpha}\ \Gamma_{(\alpha)}}\, e^{(-P_r/\beta)} \quad , \tag{24}$$

and $\overline{P} = \alpha\beta$ where $\overline{P}$ is the mean particle radius.

Lee (1975) shows that the terminal fall velocity for blowing snow particles should be calculated using Carrier's (1953) drag equations. This calculation is cumbersome but can be fit to a power law relationship for typical blowing snow conditions. Thus,

$$\omega = 1.1 \cdot 10^7 P_r^{1.8} , \qquad r^2 = 0.991 , \tag{25}$$

where P_r is expressed in metres and ω in metres per second.

The drift density of the lowest layer of suspended transport (that adjacent to the saltation layer) for a distribution of particle radii is

$$\eta_{z_1} = \int_0^\infty N f(P_r) \frac{4}{3} \pi \rho_p P_r^3 \, P\left(\frac{w\uparrow}{au^*} \geq \frac{\omega}{au^*}\right) dP_r , \tag{26}$$

where N = the number of particles in saltation per unit volume,

$f(P_r)$ = the frequency of occurrence of particle radius P_r,

ρ_p = the density of blowing snow particles, and

η_{z_1} = the suspended drift density at level z_1 (lowest level).

Now

$$N = \frac{3 \eta_{salt}}{4 \pi \rho_p} \frac{1}{\int_0^\infty P_r^3 f(P_r) dP_r} . \tag{27}$$

$f(P_r)$ is found using Eq. 24 and ω is found using Eq. 25.

Defining a transfer coefficient T_c between saltating transport and suspended transport where $T_c = \eta_{z_1}/\eta_{salt}$ and combining Eqs. 24, 26 and 27 yields

$$T_c = \frac{1}{(\alpha+2)!\beta^{(\alpha+3)}} \int_0^\infty P_r^{(\alpha+2)} e^{(-P_r/\beta)} \left[2-(2/\pi)^{.5} \int_{-\infty}^{\gamma} e^{-x^2/2} dx \right] dP_r , \tag{28}$$

where $$\gamma = \frac{1.1 \cdot 10^7 P_r^{1.8}}{au^*} .$$

For given gamma distribution parameters, Eq. 28 is a function of u*. The height of the lowest layer of suspension z_1 will be just above the mean height of saltation $\bar{h}$. For modelling purposes z_1 will be arbitrarily set 1 cm above $\bar{h}$.

The equation developed by Budd (1966) describing the vertical distribution of drift density for non-uniform blowing snow particles in terms of the gamma distribution of particles sizes can be modified to express drift density in terms of fall velocities. Thus,

$$\eta_z = \eta_{z_1}[1 + \frac{\beta_\omega}{ku*} \ln(z/z_1)]^{-(\alpha_\omega+3)} \quad , \tag{29}$$

where η_z = the suspended drift density at some height z,

k = the von Kármán constant (0.4),

β_ω = β parameter for the gamma distribution of fall velocities at z_1,

and α_ω = α parameter for the gamma distribution of fall velocities at z_1.

With η_{z_1} and z_1 known, α_ω and β_ω at height z_1 can be determined. The mean fall velocity is found using the gamma distribution of particle radii and Eq. 25, giving

$$\overline{\omega} = \frac{1.1 \cdot 10^7 (\alpha+0.8)! \overline{P}_r^{\,1 \cdot 8}}{\alpha^{1 \cdot 8} \ (\alpha-1)!} \quad . \tag{30}$$

An empirical determination of α_ω and β_ω is now possible. Assuming α = 15 for suspended snow particles (Budd, 1966), a variety of gamma distributions (various β's) for radii can be transformed to distributions for ω. Using the maximum likelihood estimator, $\hat{\alpha}_\omega$, of α_ω (Greenwood and Durand, 1960), it can be shown $\alpha_\omega = 4.8$ and $\beta_\omega = \frac{\overline{\omega}}{4.8}$. Using mean fall velocities estimated from measured profiles of drift density Radok (1968) notes consistent probabilities that the upward turbulent velocities will exceed the mean fall velocity where $P(\frac{w\uparrow}{au*} \geq \frac{\overline{\omega}}{au*}) = 0.55$ for the range 1 cm $\leq z \leq$ 3 cm. Solving Eq. 23 for $\overline{\omega}$ by numerical integration gives

$$\overline{\omega}_{z_1} = 0.66 \ u* \tag{31}$$

as an estimate of the mean fall velocity in the lowest suspended layer. Assuming α_ω = 4.8 allows us to solve for $\beta_{\omega_{z_1}}$:

$$\beta_{\omega_{z_1}} = 0.1375u* \quad . \tag{32}$$

The horizontal mass flux of suspended snow per unit area perpendicular to the wind at any height can now be found assuming the mean horizontal snow particle velocity equals the mean wind velocity (Schmidt, 1982b). Therefore,

$$Q_z = \eta_z \cdot u_z \quad . \tag{33}$$

Assuming a logarithmic profile of windspeed the total suspended mass flux is

$$Q_{susp} = \frac{u* \ \eta_{z_1}}{k} \int_{z_1}^{\infty} \ln(z/z_o)(1 + \frac{\beta_\omega}{ku*} \ln(z/z_1))^{-(\alpha_\omega+3)} dz \quad . \tag{34}$$

In the model this equation is integrated to a height of 10 m. The sum of Q_{salt} and Q_{susp} represent the total mass flux in the horizontal direction per unit width perpendicular to the flow.

Vertical Snow Transport

In the absence of data on the diffusivity of blowing snow a method of determining the vertical flux proposed by Porch and Gillette, (1977) and Shiotani and Arai, (1967) is used. For blowing snow with uniform radii

$$F_{sz} = \eta_z \, \omega_z \, , \tag{35}$$

where F_{sz} is the vertical mass flux of snow at height z.

Using Eq. 25 and integrating over a gamma distribution of P_r gives

$$F_{sz} = \frac{1.1 \cdot 10^7 \, \eta_z \, (\alpha+3.8)! \, \beta_z^{1.8}}{(\alpha+2)!} \quad . \tag{36}$$

Assuming $\alpha = 15$, β_{ω_z} for the distribution of fall velocities can be found for height z. Budd (1966) shows

$$\beta_{\omega_z} = \frac{\beta_{\omega_{z_1}}}{\left[1 + \frac{\beta_{\omega_{z_1}}}{ku*} \ln(z_1/z_2)\right]} \quad , \tag{37}$$

and $\beta_{\omega_{z_1}}$ can be found from the friction velocity. If the gamma distribution is transformed from radii to fall velocities using Eq. 25 with $\alpha = 15$ the following solution for β_z as a function of β_{ω_z} is found;

$$\beta_z = \frac{\beta_{\omega_z}^{0.556}}{53143} \quad . \tag{38}$$

Using Eq. 29 for η_z, F_{s_z} can be found from Eq. 36.

Sublimation of Snow

Schmidt (1972) has developed a method of calculating sublimation from blowing snow based on heat and water vapour transfer equations. Lee (1975) has thoroughly examined the physics of the process and recommends only slight modifications in Schmidt's model. In our application, Lee's modifications are used and in addition the effect of solar radiation on the sublimation rate has been neglected. Preliminary calculations show that this effect is negligible for Canadian Prairie winter conditions. Calculation of the sublimation rate for a single particle is made using the following expression:

$$dm/dt = \frac{2\pi P_r \sigma}{\frac{r}{\lambda T \ Nu}\left(\frac{rM}{RT} - 1\right) + \frac{1}{D \rho_s Sh}} , \qquad (39)$$

where dm/dt = the rate of sublimation for a snow particle of radius P_r,

σ = the undersaturation of water vapour in air,

r = the latent heat of sublimation ($2.838 \cdot 10^6$ J/kg),

M = the molecular weight of water (18.01 kg/kmole),

R = the universal gas constant (8313 J/kmole K),

T = the ambient air temperature (K),

λ = the thermal conductivity of air (J/(msK)),

D = the diffusivity of water vapour in air (m^2/s),

ρ_s = the saturation density of water vapour at T (kg/m^3),

Nu = the Nusselt number,

and Sh = the Sherwood number.

σ is determined using the water vapour density, ρ_w, and the saturation water vapour density, ρ_s, at temperature T. Thus

$$\sigma = (\rho_w/\rho_s - 1) \quad . \qquad (40)$$

Vapour densities are calculated from dewpoint and air temperatures using standard meteorological techniques. The thermal conductivity of air is a function of air temperature. An empirical line fit to values provided by List (1949) is

$$\lambda = 0.00063T + 0.0673 \ . \qquad (41)$$

The diffusivity of water vapour in the atmosphere from an ice sphere is also temperature dependent. The equation listed by Gray (1963) is

$$D = 2.06 \cdot 10^{-5}(T/273)^{1.75} \quad . \qquad (42)$$

The dimensionless Nusselt and Sherwood numbers have been shown by Lee (1975) to have equal values for blowing snow particles. For particle Reynold numbers, R_e, between 0.7 and 10

$$Nu = Sh = 1.79 + 0.606 \ Re^{0.5} \quad . \qquad (43)$$

The particle Reynolds number is a function of its radius, the ventilation rate V(m/s) and the kinematic viscosity of air where

$$Re = \frac{2\ P_r\ V}{\nu} \quad . \qquad (44)$$

Lee (1975) shows that V equals the particle fall velocity, ω, plus the vector sum of the root mean square (r.m.s.) relative turbulent velocity of the particle. Assuming the three cartesian components of this velocity are approximately equal

$$V = \omega + 3\omega_r \cos(\pi/4) \ , \qquad (45)$$

where ω_r is the component of the r.m.s. relative velocity. ω_r is a strong function of windspeed and a lesser function of P_r, z_o and z. Based on information in Lee (1975) and assuming z_o = 0.001 m, z = 1 m and P_r = 50 μm, an expression for ω_r as a function of u_z can be developed. Thus

$$\omega_r = 0.005\ u_z^{1.36} \quad . \qquad (46)$$

Since little is known about the structure of turbulence in the saltating layer, it is assumed that the ventilation rate for a saltating particle is the average of the mean vertical and mean relative horizontal components of the particle velocity (w_p and u_{pr} respectively). The mean vertical particle velocity $\overline{w_p}$ is assumed to be $w_{p1}/2$, where w_{p1} is the initial vertical velocity as determined from Eq. 18. The mean relative horizontal velocity $\overline{u}_{pr}$ is the mean horizontal particle velocity (Eq. 19) relative to the mean horizontal windspeed at the top of the saltation layer (where most of the horizontal component of the particle trajectory occurs). The ventilation velocity in saltation V_{salt} can be obtained by assuming a logarithmic wind profile. Thus

$$V_{salt} = \frac{\left(\frac{0.25u^* + \overline{\omega}/3}{2}\right) + \left[\frac{0.011u_1 g}{0.5u^* + 2\overline{\omega}/3} - u^*/k\ \ln(\overline{h}/z_o)\right]}{2} \quad , \qquad (47)$$

and the rate of sublimation for a single particle can be determined as a function of u_z, u*, the dewpoint temperature T_{dp} and air temperature.

For uniform conditions of u_z, u*, T_{dp} and T, the sublimation rate per unit volume of air $d\eta/dt$ is

$$d\eta/dt = N \int_0^\infty (dm/dt)_{P_r}\ f(P_r)\ dP_r \quad . \qquad (48)$$

The total sublimation rate is the sum of the sublimation occurring in the saltation layer and in the suspension layer.

Model for Fully Developed Blowing Snow

The model is demonstrated for blowing snow over fallow and unfilled stubble fields. It uses as inputs the windspeed at 10 m, the threshold windspeed for snow movement on fallow fields, the air temperature at 2 m and the dewpoint temperature T_{dp} at a 2-m height. The parameters u_{10}, T and T_{dp} are measured by AES on an hourly basis at 28 stations across the Prairies. u_{10} is used to find u*, z_o and various u_z depending on land use (fallow, stubble). In the model a neutral temperature profile is assumed. A marked decrease in relative humidity with distance from the snow surface was consistently evident during blowing snow events at Loreburn. Differences of 20% in the undersaturation of air σ, between a height of 0.05 and 2 m have been measured. The most common gradient can be simulated with the following equation:

$$\sigma_z = \sigma_{2m}[1.02 - 0.027 \ln(z)] \quad . \tag{49}$$

This equation results in relative humidity approximately 10% higher at 0.05 m than at a height of 2 m.

The threshold conditions for blowing snow are determined using hourly observations of the occurence of "ground drift" or "blowing snow" by AES. Since AES reports blowing snow over short grass or other snowfilled surfaces u_t^* is assumed to be similar to that for a fallow field. This value is used in uncorrected form although future comparisons of measurements at Loreburn with the nearby AES station may indicate the need for a correction factor.

Drift densities in saltation and suspension are plotted in Fig. 3 for a threshold u_{10} = 4.5 m/s, a typically low value and several windspeeds over fallow and stubble land. Data from Budd et al (1966) from Antarctica for u_{10} equal to 12.4 m/s and 15.3 m/s with "soft snow over hard pack snow" are also plotted. Because of the different land use and snow conditions a direct comparison between the model output and the measured data is not appropriate but the figure gives a preliminary indication that the model results are reasonable. Heights of the saltation layer are indicated by the vertically uniform drift density shown. These agree well with heights measured by Kobayashi (1972) and Kikuchi (1981). Note that the uniformity of saltation drift densities as calculated by the model does not reflect an actual condition but is a result of the modelling procedure.

The horizontal flux of blowing snow is plotted in Fig. 4 for a range of windspeeds and for various threshold conditions over fallow and stubble. As with Fig. 3 a direct comparison is not possible but these relationsips are in good agreement with data collected by Takeuchi (1980) to a 2-m height, Budd et al (1966) to a 2-m height, and Kobayashi and Yokoyama (1977 as reported by Kobayashi, 1978) to a 10-m height, for widely varying threshold conditions. The modelled flux increase follows the form reported by observers of both saltating and suspended flux (see Takeuchi, 1980). The increase for stubble land use indicates high threshold conditions with a rapid increase in mass flux once the threshold is overcome. This may be attributed to the high levels of shear exerted on surface snow particles once they are ejected from the surface.

Similar graphs can be obtained for the vertical flux of blowing snow and show a somewhat higher threshold and a more pronounced rate of increase than the horizontal flux. The sublimation rate in the 10-m column of snow has also been calculated. It has been found that an increase in air temperature from -15 to -1° C results in an almost 10 fold increase in sublimation with other factors constant. The sublimation rate is not as sensitive to threshold velocities as it is to temperature, humidity and land use conditions. Schmidt's (1982b) estimates based on profile measurement of wind, temperature, dewpoint and snow concentration up to 1 m are within the range of model results.

ADAPTATION OF THE MODEL TO AGRICULTURAL CONDITIONS

Natural and man-made impediments to blowing snow on agricultural terrain in the North American Prairies limit the area in which the snow flux can reach full development. Wooded shelterbelts, sloughs, ravines, coulees, stream valleys, irrigation ditches, railways, farm yards and major highways all obstruct the horizontal flux of blowing snow. Depending on the height and depth of the obstruction, the atmospheric boundary layer requires a certain distance downwind to develop fully. The fully developed blowing snow model can be made representative of terrain where only partially developed conditions exist by determining the rate of development of its components downwind of various boundaries.

Wooded Boundaries

Takeuchi (1980) measured the downwind increase in the horizontal flux of blowing snow from a wooded boundary with roughness elements approximately 3 to 5 m in height onto a completely snow covered swamp. His results show a hyperbolic increase in transport rate with fully developed transport established from 200 to 300 m downwind from the boundary. The vertical scale of Takeuchi's measurements is limited to 0.3 m, so 300 m may be a more representative value for the distance required to achieve the fully developed condition. A hyperbolic function which approximates the shape of Takeuchi's horizontal profiles is

$$L_c = \frac{Q_L - Q_i}{Q_{fd} - Q_i} = \left(\tanh\left(4\left|\frac{L - L_i}{L_{fd} - L_i}\right| - 2\right)\right)/2 + 0.5 , \tag{50}$$

where Q_i = the flux Q at the boundary,

Q_{fd} = the fully developed flux,

L_i = the horizontal coordinate of boundary,

L_{fd} = the horizontal coordinate at which flow is fully developed,

L_c = the distance coefficient.

The flux Q_L, which can represent horizontal mass flux, vertical mass flux or sublimation rate, at a horizontal coordinate L is therefore

$$Q_L = L_c(Q_{fd} - Q_i) + Q_i \quad . \tag{51}$$

Takeuchi's vegetation height of 3 m and $(L_{fd} - L_i)$ = 300 m suggest a fetch to boundary height ratio of 100:1. This corresponds to the ratio suggested by Taylor (1962) for atmospheric boundary layer development.

Using Eq. 51 the enhanced erosion rate, Q_e, in developing flow can be modelled using a simple mass balance of snow particles on a unit-volume increment downwind from the boundary. In Fig. 5 a transect of the erosion rate for typical mass and sublimation fluxes and a 300-m distance to fully developed flow is shown. The maximum erosion rate of 600 mg/m^2s corresponds to a removal rate of 2.4 mm/hour snow water equivalent (SWE).

Agricultural Land Use Boundaries

Modification of the blowing snow flux as it moves from one land-use area to another can also be modelled using Eqs. 50 and 51. For the case of transition from unfilled stubble to fallow the fully developed stubble flux is represented by Q_i and the fully developed fallow flux by Q_{fd}. For an exposed stubble with a roughness height 2 cm above the fallow roughness the distance required to establish fully developed flow for fallow would be 2 m. If the stubble flux is zero the area under the curve in Fig. 5 would be compressed to 2 m, resulting in scouring of most snowcovers in a matter of minutes. This zone of scour would migrate downwind of the land use boundary.

For the case of a fallow-to-stubble transition a 2-m zone of deposition would result. When the snow accumulation in this zone approaches the height of the stubble stalks (within 1 or 2 cm) the stubble would become aerodynamically similar to the fallow field. This filled-in stubble would maintain constant snowdepth as any further depositon would be eroded at the fallow erosion rate and any further erosion would cause it to act as unfilled stubble with the concomittant enhanced deposition rate. The fully developed flow for filled-in stubble would develop in approximately 1 m as flow passed from unfilled stubble to filled-in stubble. This development would result in scour of snow trapped by stubble from earlier blowing snow events. Filling-in and scouring adjacent to land use changes have been identified by Nicholaichuk et al (1985).

Agricultural Terrain Modelling

To develop representative values of sublimation rates and erosion rates for various land uses given the agricultural terrain geometry of an area, "typical" boundary patterns are used to calculate the mean rates for a land use type. Inputs to this calculation are representative lengths of field between land use changes (land use fetch) and representative distances between obstructions to blowing snow (fetch). The mean erosion and sublimation rates are point values averaged over the fetch and including development from a wooded boundary and from land-use changes at intervals equal to the land-use fetch. At this point in the model development the average distances that stubble is filled in and fallow is scoured must be assumed as no field data is available.

APPLICATION OF THE MODEL TO SWIFT CURRENT AND YORKTON

The Atmospheric Environment Service stations at Swift Current and Yorkton have fetches greater than 300 m and their measurements can be assumed valid for fully developed flow over fallow land. Air photos and topographic map interpretation provides the fetch and land use fetches shown in Table 1.

Hourly results of the model for February 1973, assuming 200 m of filled-in stubble downwind of the fallow-stubble boundary and local land use geometrics have been calculated. The mass fluxes show lower threshold conditions and therefore a less cohesive snowcover at Yorkton. However, higher windspeeds result in greater transport quantities at Swift Current. Erosion rates on fallow fields show a small difference between the two sites because of the shorter land-use and total fetches at Yorkton. The deposition rates at Yorkton and Swift Current are governed by the same factors and show greater net deposition at Swift Current. The maximum sublimation rates are similar for both sites and show an average rate of about 0.015 mm SWE/hour. The net values for fallow and stubble conditions are summarized in Table 2. Note that the threshold conditions for blowing snow on stubble were never exceeded. The mean sublimation rate on fallow is about 9% of the erosion rate for fallow at Swift Current and about 8% at Yorkton reflecting the cooler weather at Yorkton. Blowing snow resulted in a snow water difference of 4.2 mm between fallow and stubble at Swift Current but only 2.38 mm at Yorkton. This indicates that for Feb. 1973 the benefits of trapping snow at Swift Current were almost twice that at Yorkton.

CONCLUDING COMMENTS

A demonstration of the model for one month does not allow conclusions to be drawn on the potential for snow management at Yorkton and Swift Current nor does it allow an adequate evaluation of the model itself. However, the development process and the initial evaluation show that there are some aspects of the blowing snow phenomenon which require further work. The dynamics of saltation as related to the cohesion of the snowpack and surface roughness elements such as stubble require more detailed measurement and analysis. The characteristics of the turbulent structure of the wind at the saltation-suspension interface and above should be examined as well as its effect on vertical transport of suspended snow particles (blowing snow diffusivity). Profiles of temperature and humidity in blowing snow affect both transport rates and sublimation rates significantly and must be characterized in more detail from standard meteorological data. The effects of rolling and hilly terrain on blowing snow are not presently understood well enough to be characterized in a model of this type and would be a useful refinement. Ultimately, the model should allow the input of detailed agricultural land use geometry and snow trapping techniques and estimate the effects of these applications on blowing snow for "normal" winters and extreme conditions. Such information could be used in evaluating individual farm cropping arrangements, farm access and farm shelterbelt designs.

ACKNOWLEDGEMENTS

The authors thank R. Dodds and E. Hritzuk of Loreburn, Saskatchewan for allowing us to conduct the blowing snow experiments on their land. We also thank E. Wigham for typing the paper. This work was supported by NSERC operating grant No. A4364 and the Saskatchewan Research Council. This paper is an abridged version of Report No. 85-7, Division of Hydrology, University of Saskatchewan.

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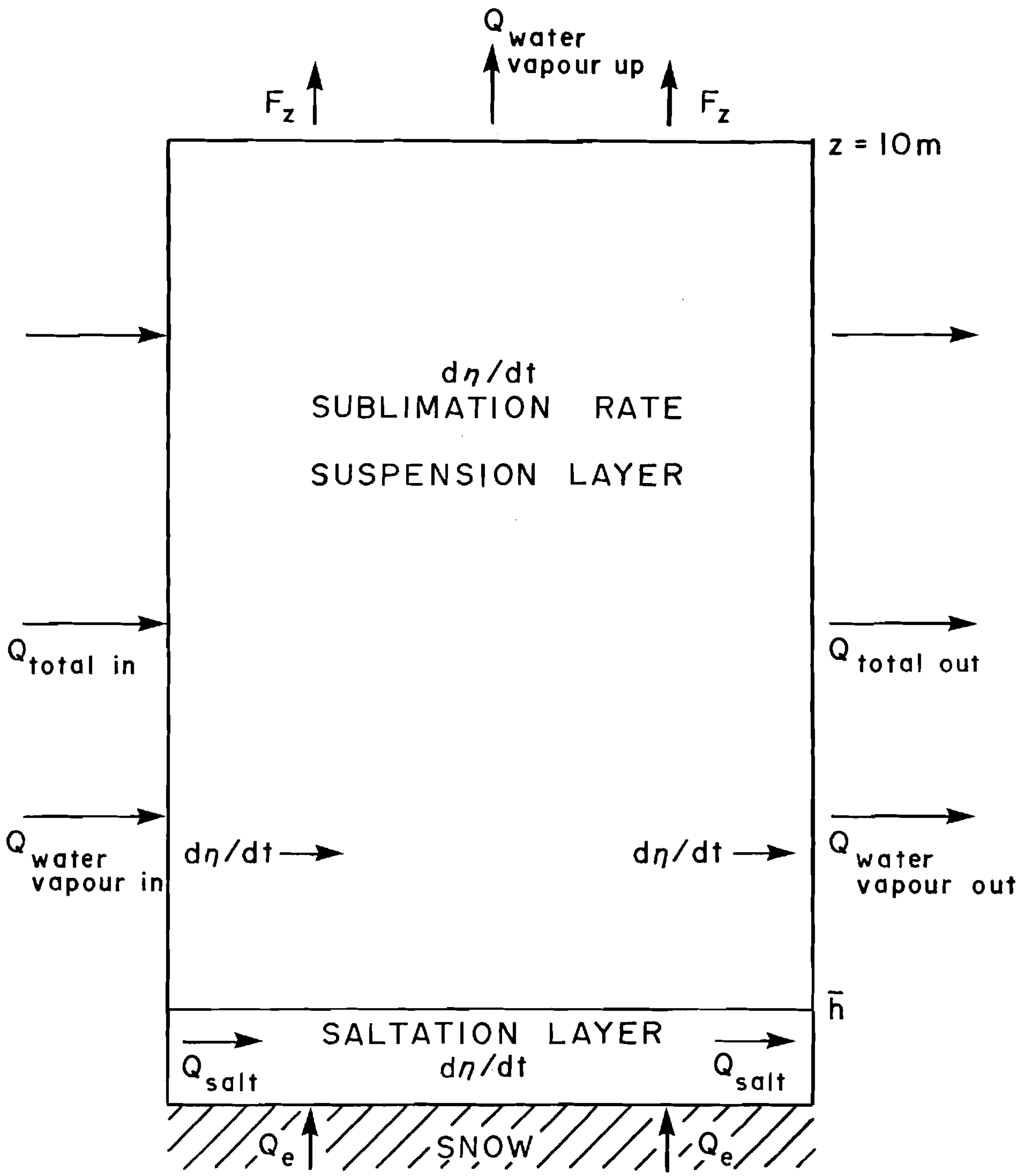

Figure 1. Control volume illustrating the solid and vapour mass fluxes in fully developed blowing snow.

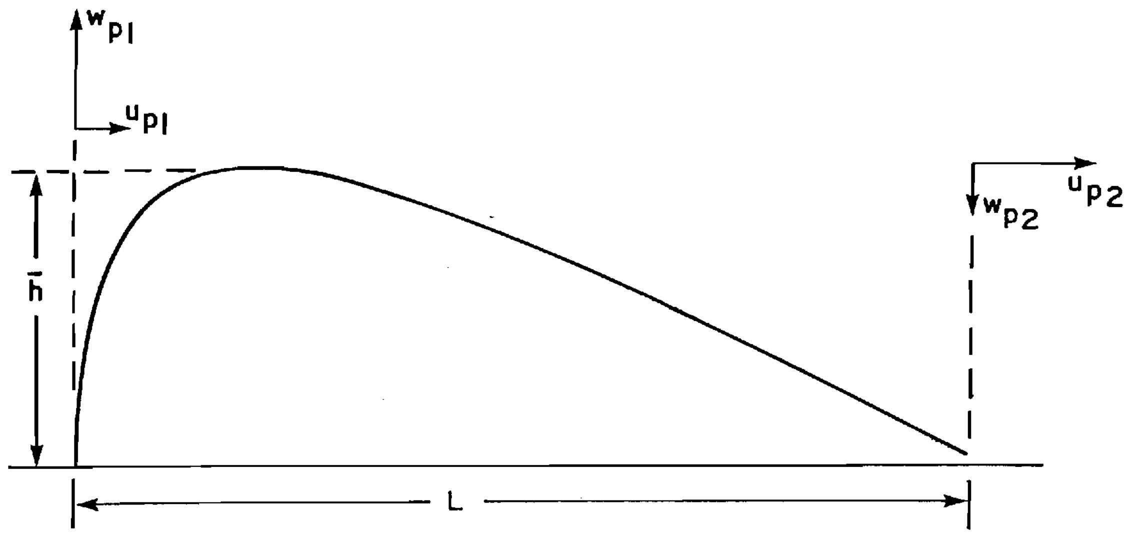

Figure 2. Initial and final velocity components of a saltating particle.

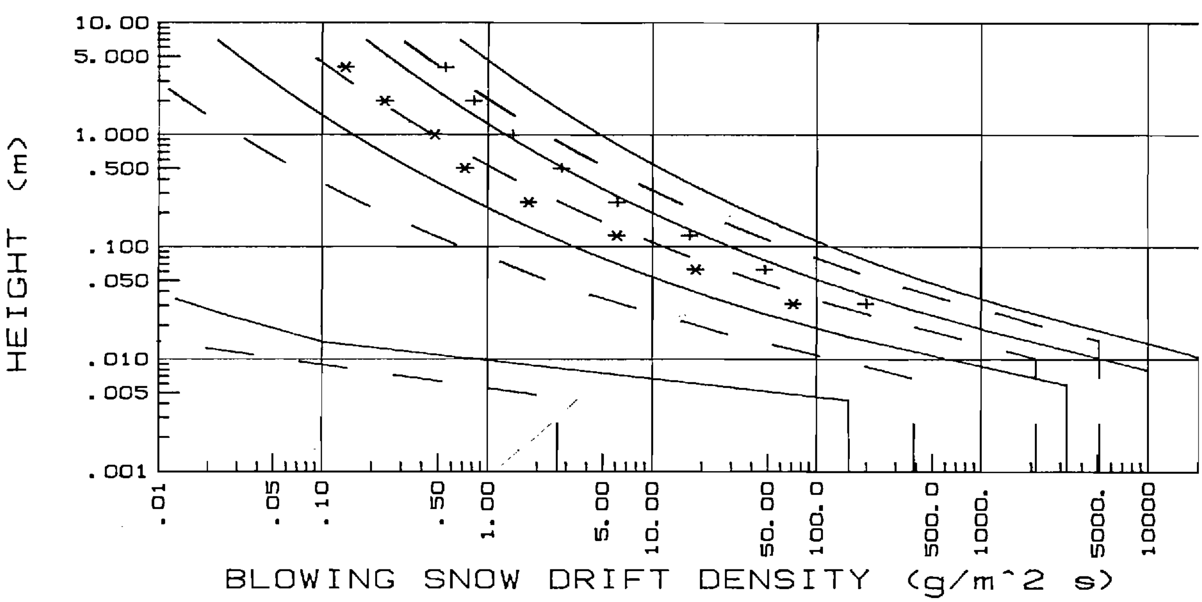

Figure 3. Profiles of drift density for stubble and fallow fields. T = -20°C, threshold u_{10} = 4.5 m/s, —— fallow, – – – stubble. Measured values are from Budd et al. (1966). * u_{10} = 12.4 m/s, + u_{10} = 15.3 m/s over soft snow.

Table 1. Representative fetch distances for Yorkton and Swift Current, Saskatchewan.

Swift Current	Yorkton
Swift Current Fetch ≃ 4000 m	Yorkton Fetch ≃ 3000 m
Fallow Land Use Fetch ≃ 1200 m (3 transitions)	Fallow Land Use ≃ 800 m (3 transitions)
Stubble Land Use Fetch ≃ 1800 m (2 transitions)	Stubble Land Use ≃ 1200 m (2 transitions)

Table 2. Net values of blowing snow parameters at Yorkton and Swift Current, Saskatchewan for stubble and fallow fields for February, 1973.

	Swift Current	Yorkton
Blowing snow	59 hrs	61 hrs
Fallow		
Mean snow water removed per m^2	2.913 mm SWE	1.63 mm SWE
Mean snow water sublimated per m^2	0.2713 mm SWE	0.135 mm SWE
Mean quantity of snow transported off the field	2465 kg/m	1070 kg/m
Stubble		
Mean snow water deposited per m^2	1.29 mm SWE	0.746 mm SWE
Mean snow water sublimated per m^2	0.029 mm SWE	0.0193 mm SWE
Mean quantity of snow transported off the field	0 kg/m	0 kg/m
Land Use Conditions		
Fetch	4010 m	3000 m
Fallow field length	1200 m	800 m
Stubble field length	1800 m	1200 m

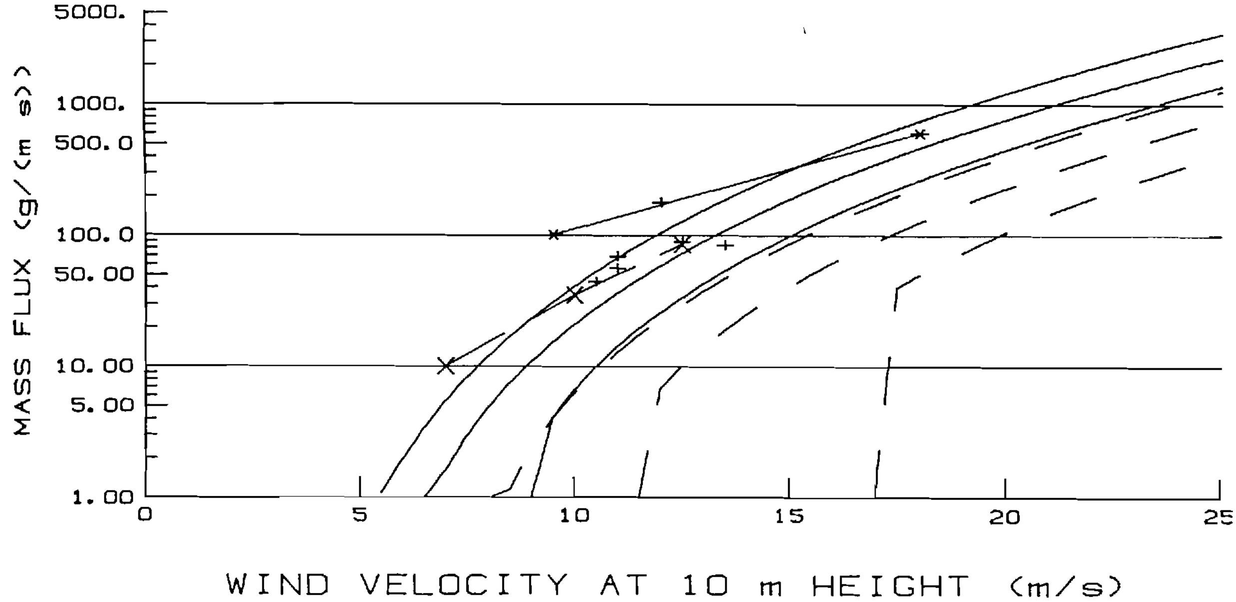

Figure 4. The horizontal flux of blowing snow to a height of 10 m for a threshold u_{10} = 4.5, 6.5 and 9.5 m/s. —— fallow, – – – stubble. Measured values are from Budd et al. (1966) to 2 m *——*, Takeuchi (1980) to 2 m x---x---x and Kobayashi and Yokoyama (1977) to 10 m +.

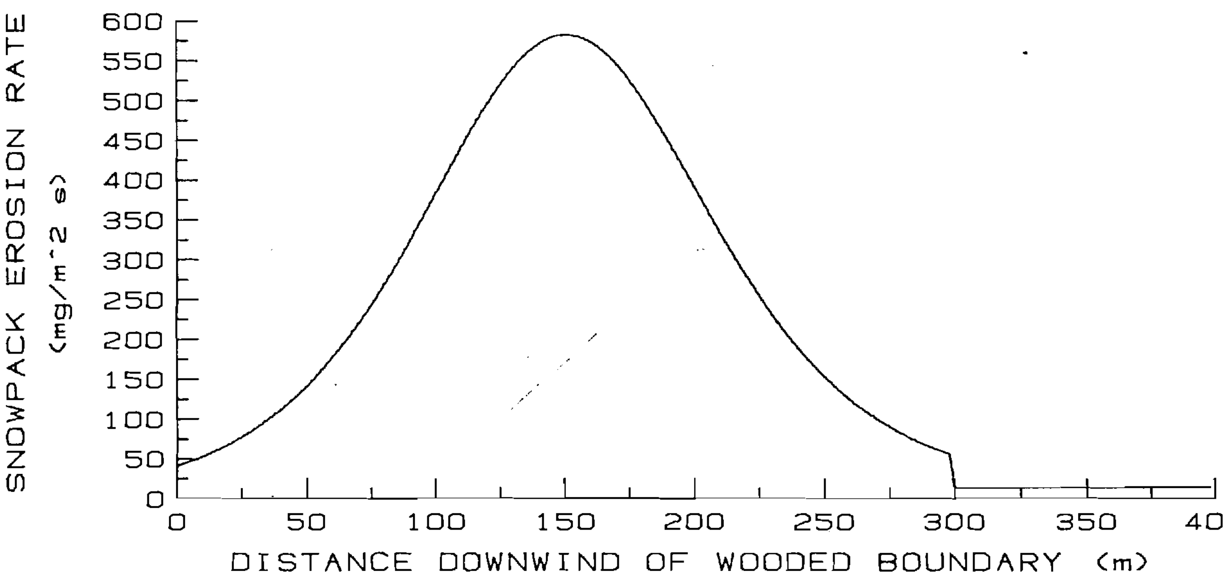

Figure 5. A horizontal transect of the erosion rate downwind of a wooded boundary assuming a fully developed horizontal blowing snow flux of 86.3 g/ms, a vertical flux of 10.68 mg/m^2s and a sublimation rate of 3.29 mg/m^2s.