

OPTICAL PROPERTIES OF BLOWING SNOW

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ABSTRACT. Calculation procedures are developed and results shown for the exact calculation of extinction and meteorological visual range using the blowing-snow mass in the atmosphere and particle radius. Results of the calculations show: (1) For monochromatic radiation, geometrical optics approximations of the extinction efficiency are found to provide results of only moderate accuracy in calculating the extinction of radiation by a single particle. (2) For broad-band radiation, the geometrical optics approximation is sufficiently accurate for many single-particle measurement instruments and applications, except in the infra-red band where Mie theory should be used. (3) For typical blowing-snow particle-size distributions, the shape parameter of the distribution of particle radii and the mean particle radius are very important in broad-band extinction and visual-range modelling. Estimates of blowing-snow quantities from broad-band extinction measurements or visual range from blowing-snow quantities should address the shape and mean value of the snow-particle radius distribution.

INTRODUCTION

The optical properties of blowing snow can have great significance to mankind's endeavour in cold regions, as any person surviving travel during a "white-out" in a blizzard can attest. These optical properties also provide a means to estimate the physical properties of blowing snow. Considering the interest by scientists and engineers in the measurement of blowing snow using its attenuation of radiation (Landon-Smith and Woodberry, 1965; Schmidt, 1977, 1984; Gubler, 1981; Pomeroy and Male, 1985) and the prediction of visual range in blowing snow (Tabler, 1979, 1984), it is appropriate to model rigorously the optical characteristics of blowing snow.

Both the visual range for an object and the transmission of light can be defined in terms of the extinction of radiation over some path of transmission. Extinction of radiation occurs as blowing-snow particles scatter and absorb the radiant flux transmitted through an ensemble of these particles. Extinction is dependent on the wavelength of radiation λ , the radius of a sphere of equivalent area to the particle r , and the cumulative area of snow particles upon which the radiation is incident along its path, $N\pi r^2$, where N is the number of particles. In visible light, the particle-size parameter $x = 2\pi r/\lambda$ is less than 50 for small blowing-snow particles. This indicates that blowing-snow particles in ambient light span the range of size from where the efficiency of extinction is variable to where it is relatively constant (van de Hulst, 1957). As extinction varies with the cumulative cross-sectional area of particles, it provides an indirect means to measure properties of blowing snow which control this area, such as the distribution of particle radii and drift density (mass of snow per unit volume of atmosphere).

Several investigations into the light-extinguishing characteristics of blowing snow have been published (Landon-Smith and Woodberry, 1965; Budd and others, 1966; Mellor, 1966; Schmidt, 1979; Tabler, 1979, 1984; Seagraves, 1981) which suggest or assume that extinction of light by blowing snow can be modelled using geometrical optics approximations. Geometrical optics assumes a uniform extinction efficiency for all particle sizes and for a beam of

parallel radiation, and specifies that twice the radiant flux which intercepts the particle is actually extinguished. Errors may accrue from assuming this constant efficiency of extinction if small blowing-snow particles or wavelengths longer than the visible range are considered. Scattering is based on spherical particles in these approximations without addressing errors due to irregularity in particle shape. The applications which attempt to relate the drift density (cumulative mass) to the cumulative cross-sectional area of the particles (Landon-Smith and Woodberry, 1965; Budd and others, 1966; Mellor, 1966; Seagraves, 1981) assume a uniform distribution of particle sizes and spherical particles. The work of Schmidt (1979) and Tabler (1979, 1984) relates the number of particles detected crossing a narrow beam of light to visual range, assuming spherical, non-absorbing particles of uniform size. Since distributions of blowing-snow particle sizes are often irregular and skewed (Budd, 1966; Schmidt, 1981), errors may accrue from assuming the mean particle mass ($r^3\rho_p$) or radius (r) corresponds to the mean particle cross-sectional area (r^2). Approaches which try to relate drift density or the number of particles to extinction should be cognizant of the magnitude of errors by:

- (1) Irregular particle shape.
- (2) Variable extinction efficiency.
- (3) Irregular distribution of blowing-snow particle sizes.

In this paper, models are presented which describe the radiation-extinguishing characteristics of blowing-snow particles in the short-wave spectrum by applying Mie's electromagnetic scattering theory (Mie, 1908; as presented by van de Hulst (1957)) for the case where forward scattering of radiation is neglected. Mie theory calculations are more accurate but much more difficult than geometrical optics approximations. The differences in the results of the two calculation procedures are discussed for cases of single particles in both monochromatic and broad-band radiation, and for ensembles of particles, so that situations where the less time-consuming geometrical optics approximations are appropriate can be defined. The implications of Mie scattering are discussed in the context of selecting wavelengths for optical snow-particle counters and relating blowing-snow drift density, and the distribution of particle radii to the transmission of radiation and the meteorological visual range.

PROPERTIES OF BLOWING SNOW

Blowing snow is defined as surface snow which has been entrained and is being transported by the wind. The dominant mode of transport near and below a height of about 0.01 m is saltation, in which snow particles, accelerated by drag with the horizontal component of the wind, periodically rebound against the snow surface in a regular exchange of momentum. Above this height, suspended transport dominates, with particles supported by the vertical component of the wind and transported by the horizontal component. Suspended blowing-snow particles have appreciable terminal fall velocities, yet can be diffused by turbulence to heights of several tens of metres when wind speeds at 10 m exceed approximately 5 m/s over exposed snow-covered terrain. Drift densities are at the maximum of

approximately 1000 g/m^3 in the saltation layer, filling one-thousandth of the atmospheric volume. Shiotani and Arai (1952), Budd (1966), and others have shown that there is an exponential decrease in the drift density as height is increased. Typically, drift densities at 2 m height are approximately 0.02 g/m^3 , filling 0.2×10^7 th of the atmospheric volume.

The size distribution of blowing-snow particles is skewed towards smaller radii and varies with height and the degree of atmospheric turbulence. It has been fitted to the two-parameter gamma distribution for both the suspended (Budd, 1966) and saltating (Schmidt, 1981) modes of transport. This distribution has the form

$$f(r) = \frac{r^{(\alpha-1)} e^{(-r/\beta)}}{\beta^\alpha \Gamma(\alpha)} \quad (1)$$

where $f(r)$ is the relative frequency of a particle of radius r , α is the distribution shape parameter, β is the scale parameter, and Γ denotes a gamma function. The particle radius is defined as the radius of a sphere of equivalent volume to that of the snow particle. The parameters α and β are also defined in terms of the mean particle radius \bar{r} (Haan, 1977) where

$$\bar{r} = \alpha\beta. \quad (2)$$

The value of α is approximately 5 for saltating snow. It increases to between 10 and 15 at a height of 2 m. The mean particle radius decreases with height, typical values being $100 \mu\text{m}$ near the surface and $40 \mu\text{m}$ at 2 m. The degree of turbulence in the wind can cause both α and \bar{r} to vary at a given height.

APPROXIMATING BLOWING SNOW AS ICE SPHERES

Blowing-snow particles are usually metamorphosed fragments of the surface snow-pack. The particles are initially shattered from the snow-pack by the impact of already entrained particles and are then abraded by further impacts during saltation. Suspended particles are derived from saltating particles which are small enough to diffuse vertically. During suspended transport, the rate of sublimation is enhanced due to high particle ventilation rates (Schmidt, 1982). This process further rounds and smooths the particles. Figure 1 is a photograph of blowing-snow particles sampled at a height of 1 m above the snow-pack and shows the metamorphosed state of these fragments. A typical shape is slightly oblong and occasionally irregular with smooth edges (Schmidt (1981) has noted this as well). Schmidt (1984) and Gubler (1981) have commented that blowing-snow particles are much more amorphous than falling-snow particles, and thus are easily distinguished from the falling crystals. As blowing-snow particles are transported by often extreme turbulent fluctuations in the wind, it is unlikely that they are oriented; the observations of Schmidt (1984) with a double-beam particle counter confirm this. The density of blowing-snow particles has been assumed approximately equal to that of ice (900 kg/m^3) (Schmidt, 1981; Pomeroy and Male, 1987), because of their metamorphosed state. Thus, blowing-snow particles approach the "optically ideal" ice sphere more closely than do other forms of snow.

The discussion in this paper of Mie theory and geometrical optics extinction of radiation by blowing snow is based on the premise that the particles behave optically as spheres of ice. Hodkinson (1963) extensively examined the influence of particle shape on scattering, as modelled in geometrical optics and Mie theory. His experiments with suspensions of variable-sized ground quartz particles show that for irregularly shaped particles greater than $1 \mu\text{m}$ in visible light, measured extinction follows an average of Mie theory extinction. In terms of geometrical optics, Hodkinson concluded that diffraction by irregular, non-absorbing, randomly oriented particles is similar to that caused by spheres of equivalent cross-sectional area. External reflection is also similar as the probability of an angle of incidence is unchanged for randomly oriented particles. However, refraction as light leaves an irregular particle shows an angular deviation from that of a spherical particle. Pollack



Fig. 1. Photograph of blowing-snow particles sampled at 1 m height. The width of the sample is 50 mm. Atmospheric conditions: air temperature -12°C , relative humidity 62%, wind speed at 1 m height 11 m/s; time and place 13.00 h, 12 February 1985.

and Cuzzi (1980) provided a correction to scattering of refracted and internally reflected radiation by non-spherical particles. This correction becomes negligible as the angle of scattering approaches zero.

Prominent atmospheric optical effects during the Canadian Prairie winter are "sun dogs" and halos around the Sun which can be observed when atmospheric ice (sharp crystal form) precipitates as crystals or as an ice fog, and "pillars" of light above street lamps which can be observed during snowfall when wind speeds are low and snow crystals are large. Over three winters of field observation in Saskatchewan, the authors have never observed sun dogs, halos, or pillars of light during blowing snow, only a diffuse appearance to the Sun or headlights of automobiles. This combination of optical phenomena is associated with light scattered by randomly oriented, non-uniform spheroids of size larger than the wavelengths of light.

Considering these observations and noting that blowing-snow particles are more spherical than Hodkinson's (1963) quartz crystals, it is suggested that the approximation of blowing-snow particles as spheres in the calculation of extinction results in small errors. This conclusion does not necessarily apply to cases where forward scattering must be accounted for.

RADIATION EXTINCTION BY BLOWING SNOW

Extinction of radiation by blowing snow may be considered for several cases: monochromatic radiation, individual particles; broad-band radiation, individual particles; and broad-band radiation, ensembles of particles. Results from examination of the first two cases are applicable to a blowing-snow particle detector which measures the drop in radiative transmittance associated with a single particle. The third case is applicable to measurement of large numbers of blowing-snow particles as in the transmissometer or optical attenuation meter. The third case can also be applied to problems of modelling visibility in blowing snow.

Individual particles, monochromatic radiation

The transmittance of radiation intercepted by a single particle is the ratio of the radiation transmitted in a forward direction after scattering and absorption by the particle to the radiation which would have been transmitted without interception by the particle. In this paper, all radiation scattered and absorbed is considered unavailable for transmission in a forward direction, i.e. no scattered radiation is detected. This is common where the angle between the particle centre and detector diameter is very small.

The extinction efficiency Q_{ext} is the ratio of the radiant flux scattered and absorbed by a particle to that

which actually intercepts the particle surface (dimensionless). The variation of the extinction efficiency with the particle radius and incident wavelength must be defined to determine the transmittance associated with a particular particle. In geometrical optics, Q_{ext} is considered equal to 2.0. This implies a quantity of radiation equal to that intercepting the particle surface is extinguished via refraction, reflection, and absorption, and an equal amount by Fraunhofer diffraction by the particle edge. This approximation is only valid for particles which are large with respect to the incident wavelength. For smaller particles, the extinction efficiency varies substantially and is a function of the particle-scattering parameter $x = 2\pi r/\lambda$ and the complex index of refraction. As some blowing-snow particle sizes lie within the range where the extinction efficiency becomes variable, it is of interest to examine the more precise Mie theory predictions of Q_{ext} . Mie (1908) employed electromagnetic theory to describe rigorously the perturbation of a planar coherent wave by spherical particles. van de Hulst (1957) detailed the Mie theory calculations of extinction efficiency for transparent, partially absorbing, and opaque spheres. The full Mie calculation is tedious and even the fastest algorithms (Wiscombe, 1980; Ungut and others, 1981) require extensive computing time, particularly when x is large and a range of size parameters is considered. Nussenzweig and Wiscombe (1980) proposed asymptotic approximations to the extinction efficiency with calculations based on Mie theory. These calculations exhibit errors of less than 0.01% for large-size parameters when compared with exact Mie calculations and are well suited to applications where an ensemble of non-uniform size particles is considered.

Warren (1984) has compiled the real and imaginary components of the refractive index of ice, for various wavelengths. Approximating blowing-snow particles as ice spheres of equivalent cross-sectional area, the extinction efficiency has been calculated for various particle sizes and incident wavelengths using Nussenzweig and Wiscombe's (1980) algorithm. The results, for monochromatic radiation, are plotted against snow-particle radius for various wavelengths in Figure 2. The wave pattern in Q_{ext} is a manifestation of interference phenomena amongst various orders of scattering rays. Figure 2 suggests that when extinction of monochromatic radiation by individual snow particles is considered (such as the case of a laser particle counter in which the detector receives no forward scattering), appreciable variation in the extinction efficiency for particles of similar size can occur. This effect is most pronounced for smaller particles and longer wavelengths, for example, a wave-pattern amplitude of greater than 15% is manifest by particles under $19 \mu\text{m}$ radius extinguishing radiation with a wavelength of $2 \mu\text{m}$.

To illustrate precisely where unacceptable differences exist between the geometrical optics extinction efficiency, $Q_{\text{ext}} = 2.0$, and the Mie theory predictions, Table I indicates the smallest snow-particle radii for which differences of less than 1–5% occur for various monochromatic wavelengths. Particles smaller than the radii listed can result in differences exceeding the stated limit. When variations from the geometrical optics extinction efficiency of 2.0 exceed 5%, the wave pattern in Q_{ext} makes relating the extinction by an individual particle to its radius extremely difficult, even with Mie theory calculations. An irregularity in particle shape will offset the wave pattern in Q_{ext} , compounding the difficulties. For differences less than 5%, calibration of a snow-particle detector using Mie theory approximations is appropriate if the errors in using geometrical optics are tolerable. An example of the application of Table I is if the designer of a monochromatic optical particle detector has set an error tolerance of 3% in his theoretical calibration, and wishes to estimate the size of particles with radii as small as $20 \mu\text{m}$, he can use the 300 nm with geometrical optics but not the 600 nm wavelength. If the designer is willing to calibrate the device with Mie theory approximations he can use the 600 nm wavelength but will exceed the 5% limit and enter essentially unpredictable ranges of Q_{ext} for the 1.06 and $2 \mu\text{m}$ wavelengths.

Individual particles, broad-band radiation

When radiation possessing a range of wavelengths is transmitted, the wave patterns shown in Figure 2 average

TABLE I. SMALLEST SNOW-PARTICLE RADII (μm) FOR WHICH THE DIFFERENCE BETWEEN THE EXTINCTION EFFICIENCY CALCULATED USING APPROXIMATIONS TO THE MIE THEORY AND THAT CALCULATED USING GEOMETRICAL OPTICS IS LESS THAN 1, 2, 3, 4, AND 5%, RESPECTIVELY, FOR MONOCHROMATIC RADIATION

% Difference	Wavelength			
	300 nm	600 nm	1.06 μm	2 μm
1	48	95	170	>300
2	22	48	84	124
3	17	30	54	87
4	6	20	37	62
5	5	12	32	53

out somewhat. Hodkinson (1963) noted that the wave patterns also average out when radiation is scattered by an ensemble of irregularly shaped particles. Broad-band radiation possesses a range of wavelengths, and its extinction by blowing-snow particles will follow the mean of the wave patterns. To determine a mean Mie extinction efficiency, a particle radius and one of four central wavelengths are used to calculate the particle-size parameter $x = 2\pi r/\lambda$. The Mie approximation of extinction efficiency is then calculated using an appropriate refractive index for ten wavelengths within $\pm x/6$ of each x and the mean of these values taken as the mean Mie extinction efficiency for the particle radius. This variation in x is equivalent to a 100 nm range of wavelength for a mean wavelength of 300 nm, increasing to a 660 nm range at $2 \mu\text{m}$. The Mie extinction efficiency for a single particle in broad-band radiation (mean Mie extinction efficiency) is shown in Figure 3 plotted against the particle radius, for four central wavelengths. Note that the smoothing of extinction efficiencies becomes less pronounced as the wavelength increases. It is clear that, even for radiation possessing a range of wavelengths, errors of a notable magnitude can occur when approximating the extinction efficiency of a single blowing-snow particle with the geometrical optics value. Thus, the total radiation extinction by a single particle is not necessarily uniquely associated with a particular radius.

Designers of optical particle counters should be aware of the variation of the extinction efficiency when selecting wavelength and band width of radiation, and in interpreting measurements by these devices. For broad-band radiation as calculated in this section, Table II shows the smallest particle radii for which the difference between the Mie theory and geometrical optics Q_{ext} are less than 1–5% respectively for various central wavelengths. While comparison of broad-band values with Table I indicates considerable improvement over the monochromatic case, there are still ranges of particle size and wavelength where the errors of using geometrical optics may exceed the design limits of snow-particle detectors. A significant advantage of broad-band radiation is that for mean wavelengths less than $2 \mu\text{m}$, the extinction efficiency can be predicted for a particle radius. For these wavelengths, selection of the calculation procedure for calibration and the central wavelength to be employed can be based entirely on the acceptable level of error for gauge calibration. For the example of the particle-detector design in the last section, a further option is to use broad-band radiation and the appropriate central wavelength as chosen using Table II.

Ensemble of particles, broad-band radiation

The electromagnetic transmittance through an ensemble of randomly located particles is the ratio of the radiant flux transmitted through this mixture to that flux transmitted without the scattering and absorption effects of the particles. This is the transmittance measured by an optical detector which receives no forward-scattered light. Following the Bouguer–Lambert law, the transmittance τ through an ensemble of particles smaller than the beam of radiation is found from the length of transmission l and the radiation

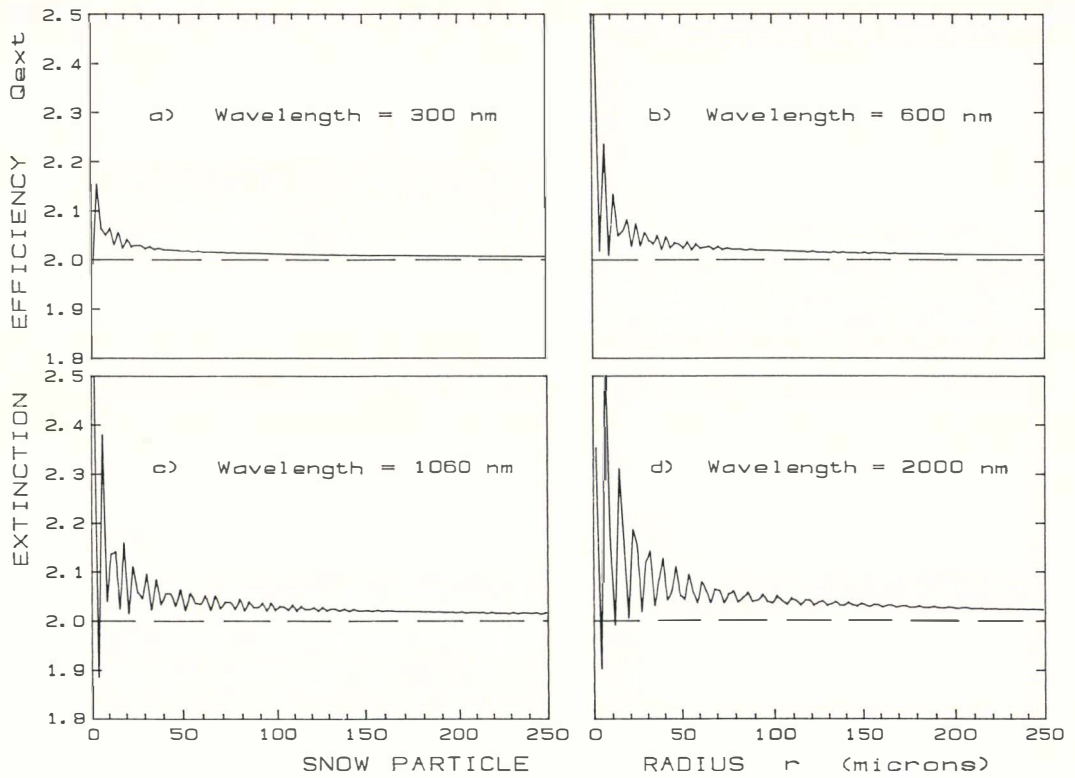


Fig. 2. Mie extinction efficiencies calculated for individual blowing-snow particles in monochromatic radiation at wavelengths of (a) 300 nm, (b) 600 nm, (c) 1.06 μm , (d) 2.0 μm . Note the geometrical optics approximation of 2.0. Wave forms are due to diffraction patterns.

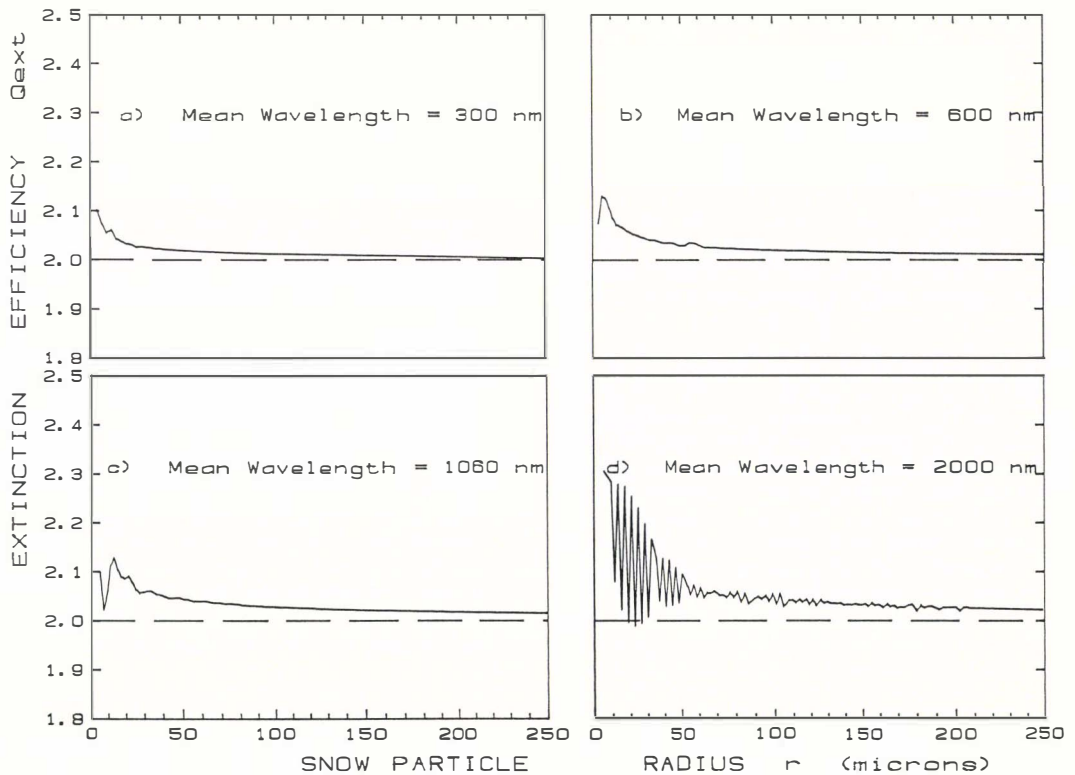


Fig. 3. Mie extinction efficiencies calculated for individual blowing-snow particles in broad-band (width = $x/3$) radiation at wavelengths of (a) 300 nm, (b) 600 nm, (c) 1.06 μm , (d) 2.0 μm . Note the subdued wave forms and closer correspondence to the geometrical optics approximation of 2.0 compared with Figure 2.

TABLE II. SMALLEST SNOW-PARTICLE RADII (μm) FOR WHICH THE DIFFERENCE BETWEEN THE EXTINCTION EFFICIENCY CALCULATED USING APPROXIMATIONS TO MIE THEORY AND THAT CALCULATED USING GEOMETRICAL OPTICS IS LESS THAN 1, 2, 3, 4, AND 5%, RESPECTIVELY, FOR BROAD-BAND RADIATION

% Difference	Wavelength			
	300 nm	600 nm	1.06 μm	2 μm
1	47	94	166	314
2	16	31	62	136
3	13	18	34	81
4	5	11	24	55
5	4	9	16	50

extinction coefficient μ_λ for the wavelength λ considered, where

$$\tau = e^{(-\mu_\lambda l)} \tag{3}$$

In this application, the small particles are particles of blowing snow, the mixture is composed of snow and air, and the length of transmission refers to distance of transmission through the atmosphere.

The extinction coefficient is the relative radiant flux which is scattered and absorbed per unit volume of the medium and has units of l^{-1} . All radiation scattered and absorbed is considered unavailable for transmission in its original direction. The extinction coefficient is a property of blowing snow and can be derived from the optical effects of individual snow particles if a quantity of blowing snow can be described in terms of these particles. Thus, for a non-uniform distribution of particle sizes,

$$\mu_\lambda = \pi N \int f(r) r^2 Q_{\text{ext}}(r, \lambda) dr \tag{4}$$

In Equation (4), N is the number of snow particles per unit volume of atmosphere, and $Q_{\text{ext}}(r, \lambda)$ denotes the particle-extinction efficiency, a function of particle radius and incident wavelength.

The extinction coefficient can be expressed in terms of the drift density, the particle-size distribution, and extinction efficiency by the following substitution. The drift density ρ_d is defined as

$$\rho_d = 4/3 \pi \rho_p N \int f(r) r^3 dr \tag{5}$$

where ρ_p is the density of blowing-snow particles, considered equivalent to that of ice. Solving Equation (5) for N and substituting this into Equation (4) yields

$$\mu_\lambda = \frac{3 \rho_d \int f(r) r^2 Q_{\text{ext}}(r, \lambda) dr}{4 \rho_p \int f(r) r^3 dr} \tag{6}$$

Including the gamma distribution (Equation (1)) to describe the frequency of particle radii, and integrating where possible over the particle radius, results in the expression

$$\mu_\lambda = \frac{3 \rho_d \int r^{\alpha+1} e^{(-r/\beta)} Q_{\text{ext}}(r, \lambda) dr}{4 \rho_p (\alpha + 2)! \beta^{(\alpha+3)}} \tag{7}$$

Thus, the extinction coefficient is a function of ρ_d , α , β , and λ .

Equations (6) and (7) assume that the radius of a sphere of cross-sectional area equal to that of a blowing-snow particle is equal to the radius of an ice sphere of mass equal to that of the particle. This approximation presumes blowing-snow particles possessing spherical shapes and densities very near that of ice. Such an approximation has proven adequate for falling-snow optical modelling (Seagraves, 1984). Until further data become available on shape factors and densities of blowing-snow

particles, correction factors cannot be confidently specified for Equations (6) and (7).

As the extinction efficiency varies within the range of blowing-snow particle sizes, precise calculation of the extinction coefficient involves defining the particle-size distribution parameters, evaluation of the Mie Q_{ext} , and solving for various wavelengths and drift densities. The work of Budd (1966) and Schmidt (1982) suggests that a gamma distribution with an α ranging from 5 to 15 is a suitable mathematical description of the suspended blowing-snow particle-radius distribution. With α fixed, the mean particle radius can be used to find the β parameter (see Equation (2)). Thus, the gamma distribution can be specified by \bar{r} and α . The blowing-snow extinction coefficient for α equal to 5, 10, and 15, mean particle radii equal to 40 and 90 μm , and a variety of wavelengths has been calculated as a function of the drift density and is shown in Figure 4. The size distributions of snow particles correspond to conditions commonly found in the suspended transport layer of blowing snow below a height of 3 m. Variation in wavelength from ultra-violet (300 nm) to infra-red (2 μm) results in an extinction-coefficient variation of 2.6%. This variation is so small that on the scale of Figure 4 it is impossible to distinguish among the lines representing different wavelengths for a particle-radius distribution. The variation in the extinction coefficient associated with various α is approximately 19%. This variation should be taken into account in any measurement application. A reduction in the mean particle radius from 90 to 40 μm results in an approximate doubling of the extinction coefficient for a constant drift density. The mean particle size (or beta parameter of the gamma distribution of particle radii) is the most important factor affecting the blowing-snow drift density-extinction coefficient relationship for the wavelengths examined. However, in interpreting measurements made using a transmissometer, values for both the alpha and beta parameters of the gamma distribution should be addressed.

VISUAL RANGE IN BLOWING SNOW

Calculation of visibility in blowing snow is an application of the extinction of broad-band radiation by ensembles of particles. Blowing snow limits visibility because snow particles scatter and absorb electromagnetic radiation. Viewed through blowing snow, the observed contrast between sources of differing luminance is reduced (scattering) along with a reduction in illuminance (absorption). The reduction of contrast limits the distance from which objects can be distinguished.

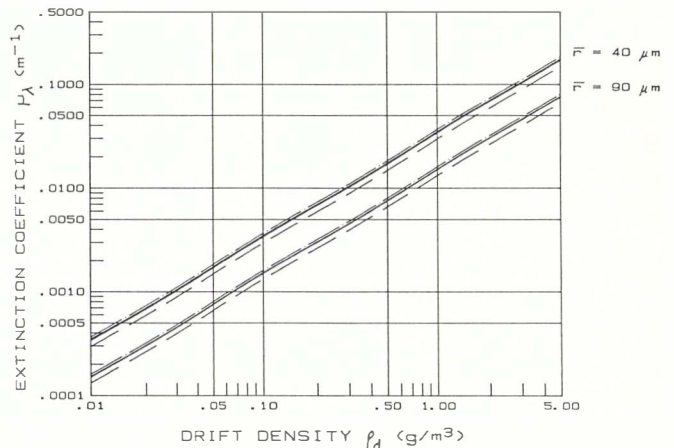


Fig. 4. Radiation extinction coefficients in blowing snow. Extinction is calculated using Mie theory for a gamma distribution of particle radii α of — — — 5, ——— 10, and — · — · — 15; \bar{r} of 40 and 90 μm ; and wavelengths from 300 nm to 2.0 μm . Data for the range of wavelengths are not readily distinguished from one another at this scale.

The meteorological visual range V is a frequently and extensively measured variable; as such it is a useful index for other blowing-snow properties. V is the maximum distance at which the apparent contrast between a black object and its white background is 2% of its inherent contrast. For meteorological measurements, this is considered to be the distance which a person with good vision can distinguish a black object occupying a 1° solid angle with his eye. The visual range can be defined in terms of the extinction of light using Koschmieder's relationship, an inverse linear function of the extinction coefficient (Middleton, 1952) where:

$$V = 3.912/\mu_{vis}, \quad (8)$$

in which V is in metres when μ_{vis} , the extinction coefficient for visible light (mean $\lambda = 600$ nm), has units of $1/m$. The extinction coefficient for a fixed quantity of blowing snow does not vary substantially over the optical wavelengths. It is therefore possible to develop Equations (7) and (8) into an operationally viable relationship between drift density and the meteorological visual range. It should be noted that there are large potential errors in specifying the meteorological visual range using Koschmieder's definition. The contrast threshold of 2% varies with the level of illumination and thresholds vary among observers. However, the meteorological visual range remains the standard atmospheric visibility measurement in Canada and other countries.

The mean visual extinction efficiency $\overline{Q_{ext}}$ is calculated by integrating the extinction efficiency at a wavelength of 600 nm over various particle-size distributions. Table III shows the mean visual extinction efficiency calculated using the Mie approximation for a particle-radius distribution α

TABLE III. MEAN PARTICLE RADII AND CORRESPONDING MEAN VISUAL EXTINCTION EFFICIENCIES. MEAN EXTINCTION EFFICIENCIES $\overline{Q_{ext}}$ ARE CALCULATED BY INTEGRATING THE EXTINCTION EFFICIENCY AT A WAVELENGTH OF 600 nm OVER A GAMMA DISTRIBUTION OF PARTICLE RADII WITH $\alpha = 10$. MEAN PARTICLE RADII \bar{r} ARE CALCULATED FROM THE β PARAMETER OF THE GAMMA DISTRIBUTION USING EQUATION (2) AND ASSUMING $\alpha = 10$

\bar{r} μm	$\overline{Q_{ext}}$
30	2.0458
40	2.0377
50	2.0324
60	2.0287
70	2.0259
80	2.0237
90	2.0219

parameter of 10 and a range of mean particle radii (note $\bar{r} = \alpha \times \beta$). The striking aspect of these results is the limited variation in the extinction efficiency for typical non-uniform distributions of blowing-snow particle sizes. For a specific particle-size distribution $\overline{Q_{ext}}$ is a constant and can be calculated as a function of the mean particle radius if the shape parameter of the radius distribution is estimated. A regression analysis on the results in Table III yields a simple, approximate relationship between the mean particle radius \bar{r} and the mean extinction efficiency $\overline{Q_{ext}}$ at a wavelength of 600 nm and a radius distribution α of 10, where

$$\overline{Q_{ext}} = 1.82 \bar{r}^{-0.011}, \quad (9)$$

and the coefficient of determination is 0.99.

Using the results of Equation (9) or the geometrical optics approximation of 2.0, Equation (7) can now be fully integrated, combined with Equation (8), and expressed in terms of the visual range, drift density, and mean particle radius, where

$$V = \frac{5.217 \bar{r} \rho_D}{\rho_d \overline{Q_{ext}} c_\alpha} \quad (10)$$

and

$$c_\alpha = \frac{(\alpha + 1)! \alpha}{(\alpha + 2)!} \quad (11)$$

The term c_α is the gamma-distribution shape-correction factor. Values of c_α vary from 0.7413 for $\alpha = 5$ to 0.8333 for $\alpha = 10$ and 0.8824 for $\alpha = 15$. For $\alpha = 10$, the per cent difference between the exact solution for the visual range as calculated using Equations (7) and (8) and that calculated using Equation (10) is less than 0.3%, the difference due to the approximation using Equation (9) which assumes $\alpha = 10$ when calculating $\overline{Q_{ext}}$. Use of the geometrical optics approximation of 2.0 for the extinction efficiency results in an added error which varies from 1% for a \bar{r} of 90 μm to 2% for a \bar{r} of 30 μm .

The meteorological visual range in blowing snow has been calculated using Equation (10) and for precision, Equation (9), for $\alpha = 10$ and a variety of mean particle radii and drift densities. The results are shown in Figure 5. The mean particle size has a significant effect on the visual range—drift density relationship. Meteorological visual range

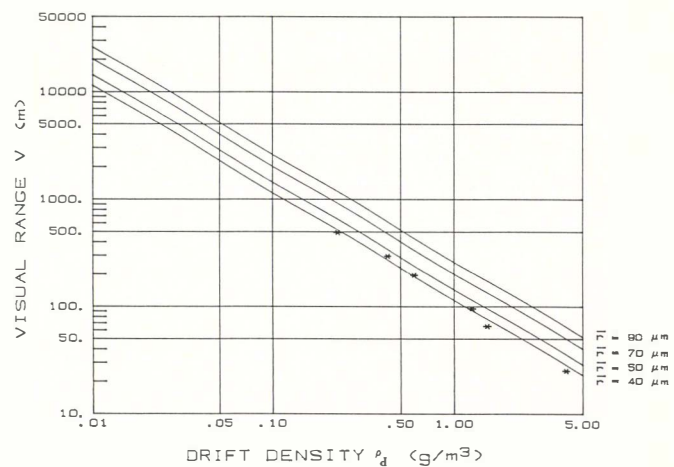


Fig. 5. Visual range in blowing snow. Extinction is calculated using an α of 10 and \bar{r} of 40, 50, 70, and 90 μm . Budd and others' (1966) observations are plotted as (*).

following Middleton's (1952) specification and the drift density at 2 m height were precisely measured in the Antarctic during blowing snow by Budd and others (1966). Their measurements are also plotted in Figure 5 and correspond to the theoretical predictions for a mean particle radius of about 40 μm . Mellor (1965) noted that the mean particle radius of blowing-snow particles at visual height (2 m) during the experiments of Budd and others is 44 μm .

DISCUSSION

Appreciable variation in the extinction efficiency of monochromatic radiation is manifest by individual blowing-snow particles of small size. For example, the extinction efficiency for blowing-snow particles in near-infra-red radiation varies irregularly over a range of about 5% for particles of less than 40 μm and about 10% for particles of less than 15 μm radius (see Fig. 2d). Shorter wavelengths exhibit an irregular variation in the extinction efficiency of a lesser magnitude. This effect must severely limit the accuracy of size estimates made from the transmittance of monochromatic infra-red radiation. For the full range of blowing-snow particle sizes (down to 20 or 30 μm radius), the amplitude of the extinction efficiency for monochromatic radiation limits the accurate interpretation of

extinction measurements to those using wavelengths of less than $1.06\ \mu\text{m}$. Monochromatic radiation measurements interpreted using geometrical optics should be limited to wavelengths less than $600\ \text{nm}$, though particle sizes typical of saltating snow permit use of the full visible band.

For radiation possessing a range of wavelengths, errors of an appreciable magnitude can occur when approximating the extinction efficiency of a single blowing-snow particle by geometrical optics. However, the errors are less than those for monochromatic radiation. These effects are most pronounced for small blowing-snow particles and long wavelengths. In an example using near-infra-red radiation (mean $\lambda = 1.06\ \mu\text{m}$), the smallest snow-particle radius for which errors in applying geometrical optics are less than 5% is $32\ \mu\text{m}$ for monochromatic radiation; for broad-band radiation the corresponding particle radius is $16\ \mu\text{m}$. A particle detector using visible light calibrated with geometrical optics experiences errors of less than 5% for particle radii down to $9\ \mu\text{m}$. However, the limiting radius for errors of less than 2% is $31\ \mu\text{m}$. As a typical mean blowing-snow particle radius at 2 m height is $40\ \mu\text{m}$, it is a matter of judgement based on the detector application whether use of the Mie theory calibration is justified. Obviously, for many particle sizes and wavelengths the geometrical optics approximation is sufficiently accurate for measurement applications. However, designers and operators of particle detectors using broad-band radiation should consider using Mie extinction values to calibrate the instruments for particle sizes and wavelengths where Table II indicates unacceptable errors in the geometrical optics approximation.

For the wavelengths examined, there is a very limited variation in the extinction efficiency of ensembles of blowing-snow particles possessing typical size distributions. A change in the wavelength from ultra-violet to near-infra-red changes the extinction coefficient by 2.6%. This magnitude of change can be neglected in most measurement and prediction applications. For a gamma distribution fit to a non-uniform distribution of blowing-snow particle radii, the extinction coefficient varies 19% as the shape parameter α varies from 5 to 15. This variation is notable and worth accounting for in the interpretation of output from transmissometers. A reduction in the mean particle radius from 90 to $40\ \mu\text{m}$ results in an approximate doubling of the extinction coefficient for a constant drift density. The mean particle size (or beta parameter in terms of the gamma distribution) is the most important factor affecting the blowing-snow drift density-extinction coefficient relationship for the wavelengths examined.

For blowing snow at "eye-level" height in ambient light, there is only a 0.3% difference between the distributed and mean Mie extinction calculation, and only a 1 or 2% difference between the mean Mie calculation and the geometrical optics approximation. Use of the geometrical optics approximation in visual range calculations should exhibit sufficient accuracy for most applications. However, the alpha parameter of the gamma distribution and mean particle radius are important in visual range modelling and should be estimated where visual range is predicted from drift density of particle-number density measurements. Budd and others' (1966) measurements of visual range can be successfully modelled when particle size is accounted for.

CONCLUSIONS

For calculations of extinction where detection of scattered radiation is not considered, the approximation of blowing-snow particles as ice spheres results in acceptably small errors for most applications.

For monochromatic radiation possessing wavelengths longer than the ultra-violet, geometrical optics approximations lead to errors of greater than 4% in calculations of extinction by single particles of sizes comprising a significant component to suspended blowing snow. Prediction of extinction for small particles becomes difficult even with Mie theory in cases where the extinction efficiency varies more than 5% from 2.0. This limits the usefulness of near-infra-red radiation in blowing-snow particle detection.

For broad-band radiation, the geometrical optics approximation is sufficiently accurate for many single-particle measurement instruments and applications. Designers

and operators of particle detectors using broad-band radiation should consider using Mie calculations to calibrate the instruments for wavelengths in the infra-red band.

For typical blowing-snow particle-size distributions, use of the geometrical optics approximations in broad-band extinction and visual range calculations exhibits sufficient accuracy for most applications. The alpha (shape) parameter and the beta parameter (scale parameter or its surrogate, the mean particle radius) of the gamma-distribution of particle radii are very important in broad-band extinction and visual range modelling. Applications which estimate blowing-snow quantities from broad-band extinction measurements or estimate visual range from blowing-snow quantities should address the shape and mean value of the snow-particle radius distribution.

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REFERENCES

- Budd, W.F. 1966. The drifting of nonuniform snow particles. In Rubin, M.J., ed. *Studies in Antarctic meteorology*. Washington, DC, American Geophysical Union, 59-70. (Antarctic Research Series 9.)
- Budd, W.F., Dingle W.R.J., and Radok, U. 1966. The Byrd Snow Drift Project: outline and basic results. In Rubin, M.J., ed. *Studies in Antarctic meteorology*. Washington, DC, American Geophysical Union, 71-134. (Antarctic Research Series 9.)
- Gubler, H. 1981. An electronic remote snow-drift gauge. *Journal of Glaciology*, 27(95), 164-74.
- Haan, C.T. 1977. *Statistical methods in hydrology*. Ames, IA, The Iowa University Press.
- Hodkinson, J.R. 1963. Light scattering and extinction by irregular particles larger than the wavelength. In Kerker, M., ed. *Electromagnetic scattering*. New York, Macmillan, 89-100.
- Hulst, H.C. van de. 1957. *Light scattering by small particles*. New York, John Wiley and Sons.
- Landon Smith, I.H., and Woodberry, B. 1965. The photoelectric metering of wind-blown snow. *ANARE Interim Reports, Ser. A(IV), Glaciology*, 1-18. (Publication 79.)
- Mellor, M. 1965. Blowing snow. *CRREL Monograph III A3c*.
- Mellor, M. 1966. Light scattering and particle aggregation in snow-storms. *Journal of Glaciology*, 6(44), 237-48.
- Middleton, W.E.K. 1952. *Vision through the atmosphere*. Toronto, University of Toronto Press.
- Mie, G. 1908. Beiträge zur Optik trüber Medien, speziell kolloidalen Metall-Lösungen. *Annalen der Physik*, 25, 377-445.
- Nussenzweig, H.M., and Wiscombe, W.J. 1980. Efficiency factors in Mie scattering. *Physical Review Letters*, 45(18), 1490-94.
- Pollak, J.B., and Cuzzi, J.N. 1980. Scattering by nonspherical particles of size comparable to a wavelength: a new semi-empirical theory and its application to tropospheric aerosols. *Journal of the Atmospheric Sciences*, 37, 868-81.
- Pomeroy, J.W., and Male, D.H. 1985. Optical properties and detection of blowing snow. *EOS. Transactions of the American Geophysical Union*, 66(46), 902.
- Pomeroy, J.W., and Male, D.H. 1987. Wind transport of seasonal snowcovers. In *Seasonal snowcovers: physics, chemistry, hydrology*. Dordrecht, D. Reidel Publishing Co. (NATO Advanced Study Institute Ser. C211.)

- Schmidt, R.A. 1977. *A system that measures blowing snow*. Fort Collins, CO, U.S. Department of Agriculture. Forest Service. (Research Paper RM-194.)
- Schmidt, R.A. 1979. Measuring visibility in blowing snow. In *Snow Removal and Ice Control Research. Proceedings of the Second International Symposium, held May 15-19, 1978, at Hanover, New Hampshire*. Washington, DC, National Academy of Sciences. Transportation Research Board, 200-07. (Special Report 185.)
- Schmidt, R.A. 1981. Estimates of threshold windspeed from particle sizes in blowing snow. *Cold Regions Science and Technology*, 4(3), 187-93.
- Schmidt, R.A. 1982. Vertical profiles of windspeed, snow concentration, and humidity in blowing snow. *Boundary-Layer Meteorology*, 23(2), 223-46.
- Schmidt, R.A. 1984. Measuring particle size and snowfall intensity in drifting snow. *Cold Regions Science and Technology*, 9(2), 121-29.
- Seagraves, M.A. 1981. *Some optical properties of blowing snow*. White Sands Missile Range, NM, U.S. Army Electronics Research and Development Command. (U.S. Army Atmospheric Sciences Laboratory Report ASL-TR-0091.)
- Seagraves, M.A. 1984. Precipitation rate and extinction in falling snow. *Journal of the Atmospheric Sciences*, 41(11), 1827-35.
- Shiotani, M., and Arai, H. 1952. A short note on the snow-storm. In *Proceedings of the Second Japan National Congress for Applied Mechanics*. Tokyo, Science Council of Japan, 217-18.
- Tabler, R.D. 1979. Visibility in blowing snow and applications in traffic operations. In *Snow Removal and Ice Control Research. Proceedings of the Second International Symposium, held May 15-19, 1978, at Hanover, New Hampshire*. Washington, DC, National Academy of Sciences. Transportation Research Board, 208-14. (Special Report 185.)
- Tabler, R.D. 1984. Using visual range data for highway operations in blowing snow. *Optical Engineering*, 23(1), 55-61.
- Ungut, A., Grehan, G., and Goesbet, G. 1981. Comparisons between geometrical optics and Lorenz-Mie theory. *Applied Optics*, 20(17), 2911-18.
- Warren, S.G. 1984. Optical constants of ice from the ultraviolet to the microwave. *Applied Optics*, 23(8), 1206-25.
- Wiscombe, W.J. 1980. Improved Mie scattering algorithms. *Applied Optics*, 19(9), 1505-09.

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