INDEX MODELS FOR PREDICTING GROUND HEAT FLUX TO PERMAFROST DURING THAWING CONDITIONS

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During the summer of 1973 a comprehensive measurement program was undertaken of the components of the surface energy balance over a tundra polygon near Tuktoyaktuk, N.W.T. It was found that under conditions where the development of an atmospheric boundary layer is incomplete, to prohibit the use of aerodynamic equations for calculating the sensible and latent heat flux components from wind, temperature and humidity data, simple index models may provide reasonable estimates of the ground heat flux. Two index models for predicting the ground heat flux during the thawing period are presented. One model is based on the use of cumulative net radiation; the other on cumulative air temperature measurements. The models are verified by measurements of ground heat flux taken at a depth of approximately 5 cm. The results suggest net radiation is the dominant energy source governing the supply of ground heat during the growth of the active layer.
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DURING THAWING CONDITIONS

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INTRODUCTION

The energy balance at the earth's surface is known to govern many natural phenomena including the rate of growth and decay of the active layer in permafrost regions. Nevertheless, it has proven extremely difficult to incorporate the surface energy balance into models designed to predict freezing or thawing conditions. Generally, either one or more of the terms in the energy equation cannot be estimated accurately as the hydrometeorological parameters required are not available.

Index models which are simple to apply and require one or two readily-available or easily-measurable input parameters are widely used in the design of engineering works. In this paper, two index models are proposed for calculating ground heat flux. It is shown that in a permafrost region where a well-developed atmospheric boundary layer is absent because of local terrain features, it may be possible to estimate the cumulative ground flux from cumulative values of net radiation or air temperature.

SITE DESCRIPTION AND INSTRUMENTATION

Data for this study were collected during the summer of 1973 at a site located approximately 18 km south of Tuktoyaktuk, N.W.T. in an area of rolling hills 15 to 30 m high. The instrument mast was located on a relatively level tundra polygon which was surrounded on the north, east and south sides by hills at distances between 75 to 150 m. A lake was situated approximately 50 m to the west. In the immediate vicinity of the mast were cracks approximately 1 m deep and 1 to 2 m wide.

At a site having such exposure conditions it is extremely difficult to establish the magnitudes of the latent and sensible heat fluxes at the ground surface. These fluxes cannot be calculated accurately by standard profile methods unless there is a well-developed turbulent boundary layer at the ground surface. The presence of cracks, hills and open water in the immediate vicinity of the instrument mast made it highly unlikely a boundary layer with measurable wind, temperature and humidity profiles would be present.

Nevertheless, in an attempt to obtain a quantitative assessment of the errors in the various terms in the energy budget which could be expected under these conditions, and to provide data in support of other studies, measurements of air temperature, humidity and wind profiles were made. A single instrument mast was used to collect profile data; air temperature was measured at the tundra surface at heights of 20, 40 and 80 cm, the dew point temperature at heights of 20, 40 and 80 cm and the wind velocity at heights of 20, 40, 80 and 100 cm. The air temperatures were measured with shielded thermistors, the dew point temperatures by means of lithium chloride dewcells, and the wind velocities by anemometers of the Rimco "miniature cup" impulse type.

The ground heat flux was measured directly using a heat flux plate. The sensor was placed approximately 5 cm below the tundra surface. Soil temperatures were measured by thermistors installed at depths of 5, 10, 15, 20 and 30 cm.

Incoming short wave and reflected radiation were measured directly using Kipp and Zonen pyranometers. Net radiation was measured with a Funk type net pyradiometer.

Details of the instrumentation are contained in Gray et al (1974). All measurements were made on a continuous basis with the aid of battery-driven field recorders.

BOUNDARY LAYER DEVELOPMENT

Before the energy budget can be incorporated into a predictive model for the surface heat flux it is necessary to establish the accuracy with which the sensible, latent and radiation fluxes can be calculated. As mentioned in the preceding section, local terrain conditions at the measurement site were such that it was unlikely a boundary layer was present within
which defined temperature, humidity and wind profiles exist. Within an established boundary layer it can be assumed the sensible and latent heat fluxes are constant with height which makes it possible to calculate these fluxes using various aerodynamic formulae (Sverdrup, 1936; Thornthwaite and Holzman, 1939; Deacon and Swinbank, 1958). Alternately, these terms can be measured directly using the eddy flux technique developed by Dyer (1961) in situations where the boundary layer is sufficiently well developed to allow the instrument to be placed at a height where it can monitor the unsteady components of wind, humidity and temperature.

A detailed analysis of the profiles along with the errors encountered in the calculation of the latent and sensible heat fluxes by aerodynamic formulae are given by Gray et al. (1974). Their findings may be summarized as follows:

1) Logarithmic wind velocity profiles existed at the site for much of the measurement period.

2) No systematic temperature differences could be detected during most of the measurement period indicating the temperature profile was non-existent or, at best, poorly developed.

3) With few exceptions, the difference between the upper and lower dewpoint temperatures represented a difference in absolute humidity of less than 0.0003 kg/kg. In other words, a humidity profile did not exist.

4) Application of aerodynamic formulae to calculate sensible and latent heat fluxes indicate differences in the order of 100% between the various expressions.

Based on these findings, it is concluded that calculations of the ground heat flux as a residual in the energy equation is not possible. In terms of model development, this implies some type of empirical index is necessary if the ground flux is to be estimated from measured atmospheric parameters. Many studies of different natural surfaces have shown that net radiation is often the dominant flux in the surface energy exchange. The absence of a well-defined boundary layer and the long hours of daylight would suggest net radiation as a major energy source to the ground heat flux at the study site. Therefore, it is appropriate to investigate the relationship between these two fluxes.

**EVALUATION OF MEASUREMENTS OF THE GROUND HEAT FLUX**

Heat flux plates are known to be sensitive to the medium in which they are immersed (Tanner, 1963). In addition, it is important to ensure there is a good thermal contact between the plate and the surrounding material. The flux plate used in the present study (Flux Transensor-dimensions 1.65 cm x 2.5 cm x 0.10 cm) was calibrated in the laboratory in wet sand rather than in situ. It was essential, therefore, to establish the validity of the heat flux measurements by an independent means. To this end the flux-plate data were compared with the fluxes calculated from the soil temperature profiles. For this analysis it was necessary to estimate the thermal conductivity of the material surrounding the flux plate.

If one assumes that the heat flow in the soil occurs mainly by conduction in the vertical direction and that the medium is isotropic and homogeneous, the energy equation assumes the form:

\[
\frac{\partial^2 T}{\partial z^2} = \frac{\partial T}{\partial t}
\]  

(1)

where \( \alpha \) = thermal diffusivity (\( \alpha = k/\rho c \) in which \( k \) = the thermal conductivity, \( \rho \) = the density and \( c \) = the specific heat), \( T \) = temperature, \( z \) = length in the vertical direction and \( t \) = time.

Van Wijk and de Vries (1963) show that, if it is assumed that at all depths the temperature varies as a harmonic function of time about the average value then a constant, \( D \), referred to as the damping depth, is related to the thermal properties of the soil and the frequency of the temperature wave by the relation:

\[
D = (2\pi\omega/\alpha)^{1/2}
\]  

(2)

where \( \omega = 2\pi/86,400 \) s\(^{-1} \) for a 24-hour period. Likewise, the phase shift \( L(z) \) of the temperature wave at a depth, \( z \), is equal to the ratio \( z/D \). Combining this equality with equation 2 yields the following expression for thermal diffusivity:

\[
\alpha = \omega(z/L(z))^{1/2}
\]  

(3)

Average values for \( \alpha \) may be obtained directly from this equation once \( L(z) \) is determined from the soil temperature data.

The diffusivities calculated from equation 3 at depths of 5 cm and 15 cm were 6.32 cm\(^2\)/h and 7.64 cm\(^2\)/h respectively. These values are substantially lower than those listed for mineral soils of various moisture contents and approach the average value for water in the observed temperature range (4.6 cm\(^2\)/h). Gravimetric samples of the medium were taken directly beneath the surface at a depth between 4 and 7 cm. Although no tests were conducted to determine the organic content of these samples, visual inspection indicated that they were highly organic. The samples had moisture contents in excess of 200 percent by weight, an average heat capacity of 3.60 kJ/kg·°C and an average dry density of 700 kg/m\(^3\). In the light of these results, it was
considered reasonable to assume the density of the soil was close to that of water (1000 kg/m³).

An estimate of the daily heat flux may be obtained in principle by measuring the instantaneous temperature gradient at 5 cm and multiplying by the value for thermal conductivity and integrating over a 24-h period. In practice this procedure is often unreliable because it is extremely difficult to establish the temperature gradient near the surface where large fluctuations in temperature are the rule. In view of this difficulty, the following energy balance approach was used to calculate the heat flux at the 5 cm depth.

\[
\text{Heat Flux (5 cm)} = \text{Heat Flux (17.5 cm)} + \\text{Change in Internal Energy between 5 and 17.5 cm} \tag{4}
\]

The depth of the lower boundary was chosen at 17.5 cm because an examination of the temperature data revealed that in the depth increment between 15 and 20 cm a linear temperature gradient could be used to approximate the temperature profile. Within this depth increment the gradients did not change significantly over periods of one hour making it possible to further simplify the calculations by assuming a steady state situation over this period. Thus the hourly heat flux \(q\) at 17.5 cm was calculated by assuming \(q = -k\Delta T/\Delta z\) where \(\Delta T/\Delta z\) is the average of the gradient at the beginning and end of the hour. Thermal conductivity \(k\) was calculated from the expression:

\[
k = \alpha cp
\]

and the change in internal energy in the layer between 5 and 17.5 cm \((\Delta U)\) by,

\[
\Delta U = pc\Delta z\Delta T
\]

where:

- \(\alpha = 7.64\ \text{cm}^2/\text{h},\)
- \(c = 3.60\ \text{kJ/kg°C},\)
- \(\rho = 1000\ \text{kg/m}^3,\)
- \(\Delta z = 12.5\ \text{cm and}\)
- \(\Delta T = \text{the temperature change over one hour measured at a depth of 10 cm.}\)

Figure 1 compares the measured and calculated daily heat fluxes at 5 cm for the period of study. The agreement in the time response of the measured and calculated fluxes are good with the measured values generally tending to be consistently greater than the calculated values except in late August. It is also evident that the larger differences are associated with the higher fluxes. One would expect the averaging procedure to give a low estimate of the heat flux. Also, it is possible that solar radiation penetrated to the flux plate during periods of intense radiation causing it to read high.

![Figure 1. Comparison of Measured and Calculated Daily Ground Heat Fluxes: \(\rho = 1000\ \text{kg/m}^3, k = 0.764\ \text{W/m°C}.\)](image)

Figure 2 shows the cumulative curve of measured flux plotted against the values calculated from the temperature profile data assuming; \(\rho = 1000\ \text{kg/m}^3\) and \(k(17.5\ \text{cm}) = 0.76\ \text{W/m°C}.\) The linear association between the two parameters was calculated as:

\[
\Sigma Q_G(\text{HFP}) = 1.50 + 1.16 \Sigma Q_G(\text{TP}) \tag{5}
\]

where \(Q_G(\text{HFP}) = \text{heat flux measured by the plate (MJ/m²)}\) and \(Q_G(\text{TP}) = \text{heat flux calculated by the temperature profile data (MJ/m²)}.\)

![Figure 2. Comparison of Measured and Calculated Cumulative Ground Heat Fluxes (MJ/m²).](image)
The coefficient of determination ($r^2$) for the regression is 0.999. Equation 5 suggests that on the average the measured heat fluxes were 16 percent higher than the calculated values; this constant percentage (linearity) would suggest the value of $k$ used in the calculation is low. Considering the assumptions involved in the calculations and that the heat flux plate had been calibrated it is assumed the agreement is sufficiently close to validate the measured heat flux values. Further analysis is based on the premise that the measured heat fluxes are accurate.

**DAILY VALUES OF GROUND HEAT FLUX AND NET RADIATION**

An examination of the daily changes in ground heat flux and net radiation showed that, on the average, the peak intensity of ground heat flux lagged the peak of net radiation by two hours. This lag was taken into account in calculations of the daily totals of the two components. Net radiation was calculated from the cross-over or zero point in the morning which occurred at approximately 0300 hours. The corresponding ground heat flux values were determined from 0500 hours. Both values were assigned to the same calendar day.

Figure 3 shows the daily values of $Q_G$ and $Q_N$ plotted for the measurement period. From the figure it can be observed that the response of the two variables is fairly consistent both in time and direction although the relative magnitudes of the two fluxes vary widely. Further attempts were made to relate the daily values of net radiation and ground heat flux stratified according to rain and rainfree periods. The scatter diagrams resulting from this analysis indicated the association between the variables was very weak for both situations.

Wide scatter of the daily values might be expected since the relative magnitudes of net radiation, latent and sensible heat fluxes may vary widely depending on local atmospheric conditions. Because of the large scatter, "best-fit" curves derived from these data could not be used as reliable predictive relationships.

**CUMULATIVE VALUES OF GROUND HEAT FLUX AND NET RADIATION**

From the point of view of model development the ground heat flux for any given 24 h period is probably unimportant when considering changes in the depth of the active layer or thermokarst development. Further, the total or integrated heat capacity of the active layer would tend to dampen short term variations in the surface energy flux. These considerations suggest that cumulative values of net radiation may provide a more useful and accurate index of the ground heat flux. Figure 4 shows the cumulative values of ground heat flux ($Q_G$) plotted against the cumulative values of net radiation ($Q_N$). Two features are obvious:

1) The association between $Q_G$ and $Q_N$ for rain and rainfree periods are distinctly different.
2) Within either a rain or a rain-free period the slope of the curve is relatively uniform.

During rain events the change in $Q_G$ corresponding to a given change in $Q_N$ is much smaller than for rain-free periods. Several factors may contribute to this phenomenon. The presence of rain water in the upper soil layer will temporarily increase its heat capacity. Furthermore, this water will have a more or less uniform temperature and will considerably reduce the temperature gradient at the surface and hence the heat flux into the soil. It should be recognized, however, that the result does not imply that the total energy flux to the ground is less during rain periods. Heat...
flux plates do not measure heat transferred by mass moisture movement and considerable amounts of energy may be added (advected) to the soil by the infiltrating water. In other words, the total ground flux will be the sum of the heat flux by conduction and the energy flux associated with the water movement.

On the basis of the results given in Figure 4, linear regressions between $\Sigma CQ$ and $\Sigma QN$ were developed for both rain and rain-free periods (see Figures 5 and 6). The regression equations are:

For rain-free periods

$$\Sigma Q_G = -3.94 + 0.347 \Sigma QN \quad (\text{MJ/m}^2) \quad (6)$$

$$r^2 = 0.992$$

For rain periods

$$\Sigma Q_G = -0.536 + 0.153 \Sigma QN \quad (\text{MJ/m}^2) \quad (7)$$

$$r^2 = 0.978$$

The high values of the coefficients of determination ($r^2$) suggest that 98 to 99 percent of the variation in $\Sigma Q_G$ can be explained by the linear regression with $\Sigma QN$. Although the data exhibits a sinusoidal variation about the regression line, measurement errors are such that the use of a higher order polynomial cannot be justified.

The results indicate that over periods longer than a day radiation is the dominant process contributing to the ground heat flux. There are two possible explanations for this: either the sensible and latent heat transfer processes compensate each other so that their algebraic sum is small or each of these transfer processes is unimportant in relation to the radiation exchange process. The latter possibility is more likely considering the lack of well-developed humidity and temperature profiles at the field site.

The correlation for rain-free periods suggest that the ground heat flux at a depth of 5 cm is on the average 35 percent of the net radiation. This value would appear to be unusually high considering the low conductivity and good insulating properties of the organic surface, and could only be possible if there was a large temperature difference between the tundra surface and the air.

During the measurement period, surface temperatures recorded by a shielded thermistor exceeded the air temperature at 20 cm by 6 to 8°C on clear sunny days. Balobaev (1964) has shown that under short grass conditions temperature differences in the order of 5°C are necessary to sustain a ground heat flux of the magnitude encountered in this study.

An analysis of air temperature and net radiation data revealed there was a fairly good degree of association between these two parameters ($r^2 = 0.69$). Air temperature is a much easier parameter to measure than net radiation and therefore an attempt was made to relate $\Sigma Q$ to the cumulative mean daily air temperature measured at 20 cm above the tundra surface. These data are plotted in Figure 7. The regression equation is:

$$\Sigma Q = 5.32 + 0.334 \Sigma T - 0.0002(\Sigma T)^2 \quad (8)$$

where $\Sigma Q$ has the units of MJ/m² and $\Sigma T$ has the units °C·day.
CONCLUSIONS

The results of the study suggest that index models based on cumulative net radiation or cumulative air temperature may be used to predict ground heat flux to permafrost under thawing conditions. Because of their simplicity and potential in engineering applications further study and development of these models is warranted.

REFERENCES


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