An energy-budget snowmelt model for the Canadian Prairies

D. M. Gray and P. G. Landine

Division of Hydrology, University of Saskatchewan, Saskatoon, Sask., Canada G7N 0W0

Received August 10, 1987
Revision accepted December 8, 1987

Meltwater released by shallow snow covers of the Canadian Prairies is an important water resource to the region. Therefore, many water-management agencies are interested in methods of forecasting streamflow and seasonal water yield from snowmelt. Reliable, accurate forecasts, however, require information of the time of melt and snowmelt rates and volumes. At the present time these quantities are usually estimated by simple temperature-index methods, which have not proven successful in open grassland environments.

The paper describes the development and testing of a snowmelt model that uses the energy equation as its physical framework. Empirical procedures for evaluating radiative, convective, advective, and internal-energy terms from standard climatological measurements are presented. Algorithms for accounting for changes in the energy terms in a daily energy-balance model are described.

The application of the energy-budget snowmelt model (EBSM) for predicting ablation and simulating streamflow from small and large watersheds is evaluated. It is demonstrated that the EBSM is workable in an operational forecast system and when incorporated within such a system leads to general improvement in synthesizing streamflow from snowmelt.

Introduction

Snow is an important water resource in the semi-arid region of the Canadian Prairies. Although only approximately one third of the annual precipitation occurs as snow, it produces 80% or more of the annual surface runoff. Meltwater derived from the shallow snow cover, has many beneficial uses: as a domestic and livestock supply, as a wildlife habitat, for recharging soil water reserves, and other purposes; conversely, runoff may cause localized flooding, soil erosion, and drainage problems. Thus, for different reasons, agencies concerned with water management have an interest in improved methods of predicting rates and volumes of snowmelt.

Knowledge of the time and amount of water released by snowmelt is essential for accurate short-term (1 day or 1 week) forecasts of flood flows and for reliable long-term forecasts of seasonal water yield and the soil-moisture status at the time of seeding of annual crops. Numerous models have been developed for forecasting streamflow. Included among the better known systems are the U.S. National Weather Service River Forecasting System (U.S. Department of Commerce 1972; Anderson 1973; Peck 1976); the U.S. Corps of Engineers Streamflow Synthesis and Reservoir Regulation (SSARR) model (U.S. Army Corps of Engineers 1972); the U.S. Department of Agriculture Hydrograph Laboratory model (Holtan et al. 1975); and the HBV of the Swedish Meteorological and Hydrological Institute (Bergstrom 1979). Most of these, and others have been developed from data collected in mountainous or forested terrain, in areas with maritime climate, and in physiographical regions having deep snowpacks. Hence, their applicability for predicting discharge from prairie watersheds may be questioned simply on consideration of differences in snow cover and topographic, climatic, vegetative, and land-use factors.

Phenomenologically, the conservation of energy is the basis of all snowmelt models. However, the relative importance to melt of the energy terms and transfer mechanisms varies widely with climate, vegetation, topography, and other factors. For example, a forest canopy absorbs long- and short-wave radiation and emits long-wave radiation, thus controlling the net radiative exchange and the ambient air temperature at the underlying snow surface. Because the net long-wave exchange and the convective transfer of sensible energy are temperature dependent, temperature-index methods for calculating snowmelt rates have proven successful in this environment (Price and Pentzold 1984). In open grassland environments the amount of atmospheric radiation absorbed by a snow cover is governed principally by the intensity of the incident short-wave beam and surface albedo; the net long-wave exchange process affects melt primarily by controlling the internal-energy status of a snow cover. Meltwater is not released until the energy deficit of a snowcover is satisfied and it becomes isothermal at 0°C. Further, in open areas the relative magnitudes of net radiation and sensible heat, the fluxes that govern melt, can vary widely in time and snow-cover conditions. For example, Granger et al. (1978) gave ratios for these fluxes of 0.33 and 2.67 on consecutive days of melt of a complete snow
cover. Because solar radiation is often the dominant flux, particularly under complete snow cover, and the radiative and convective energy exchanges are poorly correlated, temperature-index models of snowmelt tend to perform poorly in a prairie environment.

O'Neill (1972), Male and Gray (1975), Granger (1977), Granger et al. (1978), and Male and Granger (1979, 1981) have studied the energetics of melting prairie snow covers. Their results suggest that because of the dynamic character of the energy terms, the small energy and mass storage capacities of the shallow snow cover, the short ablation period, and other factors, reliable estimates of snowmelt are probable only if calculated within the physical framework of the energy equation applied to periods not longer than a day. In this paper, an energy-balance snowmelt model (EBSM) that can be used for forecasting streamflow from shallow prairie snow covers is developed and tested. To ensure that the model can be applied to operational problems, it includes procedures for estimating the energy terms from daily measurements of precipitation, sunshine hours, air temperature, relative humidity, and 10 m wind speed. The performance of the model in predicting snow-cover ablation and streamflow is evaluated.

Model development

Energy equation

The physical framework of the model is the energy equation, which expresses the energy available for melting a unit volume of snow cover \( Q_{SN} \) as

\[
Q_{SN} = Q_{SN} + Q_{H} + Q_{C} + Q_{E} + Q_{P} - du/dt
\]

where \( Q_{SN} \) is the net short-wave radiation flux absorbed by the snow cover; \( Q_{SN} \) is the net long-wave radiation flux at the snow-air interface; \( Q_{C} \) is the convective sensible heat flux between the air and the snow surface; \( Q_{E} \) is the convective flux of latent energy (evaporation, sublimation, condensation) at the snow-air interface; \( Q_{P} \) is the flux of energy across the snow-ground interface by conduction; \( Q_{E} \) is advective energy flux from rain; and \( du/dt \) is the rate of change in internal (stored) energy. The amount of melt can be calculated from \( Q_{SN} \) by the expression

\[
M = Q_{SN} / (\rho \cdot h_f \cdot B)
\]

where \( M \) is the depth of snowmelt; \( \rho \) is the density of water; \( h_f \) is the latent heat of fusion; and \( B \) is the thermal quality of snow or the fraction of ice in a unit mass of wet snow. For normal conditions, \( h_f = 333.5 \text{ kJ/kg}, \rho = 1000 \text{ kg/m}^3 \), and \( B \) is usually taken in the range 0.95–0.97.

Evaluation of energy terms

Expressions for evaluating the energy terms ([1]) were developed from field observations collected on the Bad Lake watershed, which is located in the semi-arid region of southwestern Saskatchewan (latitude 51.32°N, longitude 108.42°W). The data include 14 years (1972–1985, inclusive) of climatological measurements; measurements taken during a comprehensive study of the energetics of melt of shallow snow covers with a snow lysimeter (Granger 1977; Granger and Male 1978; Male and Granger 1979), and the results from snow-hydrology investigations reported by O'Neill (1972) and Male and Gray (1975).

Net radiation

Net radiation \( Q_{RN} \) is the total radiative flux absorbed by a snow cover and is equal to the sum of the net short-wave and net long-wave fluxes, that is, \( Q_{RN} = Q_{SN} + Q_{LN} \). During its history a snow cover is subjected to a wide range of net radiative fluxes caused by changes in solar angle, cloud cover, albedo, daylight hours, air temperature, humidity, and other factors. Generally, in northern latitudes long-wave flux dominates the radiative process during winter and early spring, whereas in late spring and summer the net short-wave flux is most important. This is demonstrated in the plots of measured daily net radiation and net short-wave radiation shown in Fig. 1. Figure 1a shows poor association (correlation coefficient \( r = 0.08 \)) between \( Q_{RN} \) and \( Q_{SN} \) during "premelt," the period extending from February 1 of a given year to the start of the period of rapid melt that leads to complete ablation of the seasonal snow cover (refer to Fig. 2). The strong influence of the long-wave exchange on \( Q_{SN} \) is evinced by the large number of negative fluxes, which can be attributed to the low fluxes of global radiation incident to the snow cover, the high surface albedo, low ambient temperatures, the short number of daylight hours, and clear sky conditions.

Conversely, as shown in Fig. 1b, a strong linear relationship \( (r = 0.89) \) exists between \( Q_{RN} \) and \( Q_{SN} \) during melt—the period of rapid ablation leading to the disappearance of the annual snow cover. The strong association can probably be attributed to the high levels of net short-wave radiation and the fact that rapid melting of a prairie snow cover often occurs on clear days following nights when the nocturnal long-wave loss is small or the long-wave flux positive. Interestingly, the finding is similar to results reported by Davies (1965, 1967), Davies and Idso (1979), and Mudieare (1985) for the summer months, which show that net short-wave radiation is the dominant flux to net radiation over bare and vegetative surface and that plots of \( Q_{RN} \) and \( Q_{SN} \) are remarkably linear, with high correlation coefficients.

Based on the above, different approaches were used to calculate \( Q_{SN} \) in the two periods. For premelt, an expression for \( Q_{SN} \) was developed using functional forms of relationships between \( Q_{SN}, Q_{LN}, \) and climatological variables reported by other investigators (for example, see Brunt 1932; Penman 1948; Mateer 1955; U.S. Army Corps of Engineers 1956; de Jong 1973; Satterlund 1979; Brutsaert 1982). The regression equation for daily estimates of \( Q_{SN} \) derived from analysis of the Bad Lake data by Gray and Landine (1987a) is

\[
Q_{SN} = -0.04 + 0.76 \{Q_{SN}[0.52 + 0.52n/N](1 - A) + 0.97\sigma T_a^4[(-0.39 + 0.093e_a^{-0.5})(0.26 + 0.81n/N)]\}
\]

where \( Q_{SN} \) is the daily clear-sky short-wave radiation incident to the surface \( (\text{MJ/m}^2 \cdot \text{day}) \); \( n \) is the number of hours of bright sunshine in the day \( (h) \); \( N \) is the maximum possible hours of bright sunshine in the day \( (h) \); \( A \) is the mean surface albedo; \( \sigma \) is the Stefan–Boltzmann constant \( (4.899 \times 10^{-8} \text{ MJ/m}^2 \cdot \text{K}^4 \cdot \text{day}) \); \( T_a \) is the mean daily air temperature \( (\text{K}) \); and \( e_a \) is the mean daily vapour pressure of the air at \( T_a \) (mbar).

Equation [3] has a correlation coefficient of 0.75 and a standard error of estimate of 1.01 \( \text{MJ/m}^2 \cdot \text{day} \).

For the melt period, \( Q_{LN} \) is calculated as a linear function of the daily net short-wave radiation by the expression

\[
Q_{LN} = -0.53 + 0.47Q_{SN}[0.52 + 0.52(n/N)](1 - A)
\]
Equation [4] has a correlation coefficient of 0.87 and a standard error of estimate of 1.55 MJ/m²·day.

Equations [3] and [4] require daily measurements of $T_a$, relative humidity or dew-point temperature, and $n$; values of $N$ available from tables; and estimates of $Q_o$ and $A$. The clear-sky flux is calculated by procedures proposed by Garnier and Ohmura (1970) for the direct component and by List (1968) for the diffuse component. These models require information on the solar constant, radius vector of the Earth’s orbit, hour angle measured from solar noon, slope, extraterrestrial flux, sun-zenith distance, optical depth, mean transmissivity of the atmosphere, and amount of diffuse radiation absorbed by water vapour and ozone. All atmospheric terms except the mean transmissivity and the absorbed diffuse radiation are fixed in space by geographical position and time of year, and their respective values can be either calculated or obtained from handbooks (for example, see List 1968). A mean transmissivity of 0.85 was obtained by comparing short-wave radiation measured on clear days between February 1 and April 30 with $Q_o$, calculated by Garnier and Ohmura’s (1970) model. This value is in agreement with a transmissivity of $0.80 \pm 0.05$ recommended by Kuz’min (1972) for the snowmelt period in the USSR. Following List (1968), the amounts of scattered radiation absorbed by water vapour and ozone were assumed to be 7 and 2%, respectively.

Snow-cover albedo is estimated in the model using the simulation described by Gray and Landine (1987b). The procedure assumes that the albedo depletion of a shallow snow cover, not subjected to frequent melt events, can be approximated by three line segments of different slope describing the periods premelt, melt, and postmelt (the period immediately following disappearance of the snow cover). An algorithm of the model makes use of daily inputs of net radiation (calculated by [3] and [4], maximum and minimum air temperature, a threshold air temperature of melt, and snow-cover and snowfall depths) to establish the “start” of melt and albedo depletion. Figure 2 provides a comparison between measured and simulated albedo for 1980.
Convective-energy flux

In the absence of advection from external air masses, the convective fluxes of sensible ($Q_h$) and latent ($Q_e$) energy are often of secondary importance to net radiation during the early stages of melt when snow cover is complete. For example, Granger (1977) found that diurnal evaporation was more or less compensated for by condensation at night. However, as a snow cover becomes discontinuous and energy generated by bare ground is advected to adjacent patches of snow, convection may play an increasingly important role in melt.

The fluxes of sensible and latent energy are governed by turbulent exchange processes occurring in the 2–3 m layer of atmosphere immediately above the snow surface. Many established procedures, for example, eddy correlation methods, analysis of atmospheric profile measurements, and other methods, can be used to estimate the components under complete, continuous snow cover, however, the processes involved in advection and the transfers of heat and mass to patches of snow are poorly understood, and methods of calculating average melt rates over large areas are nonexistent. The constraints of limited data, the lack of understanding of energy-transfer mechanisms under patchy snow cover, and the requirements for an operational system necessitated the use of simple, bulk equations for evaluating the terms. The following empirical expressions, derived from estimates of the convective fluxes calculated by Granger (1977) from detailed profile measurements, were used.

Sensible energy:

$$[5] \quad Q_h = -0.92 + 0.076U_{10} + 0.19T_{MAX}$$

Latent energy:

$$[6] \quad Q_e = 0.080(0.18 + 0.098U_{10})(e_s - e_2)$$

where $Q_h$ is the sensible energy (MJ/m²·day); $U_{10}$ is the mean daily 10 m windspeed (m/s); $T_{MAX}$ is the daily maximum air temperature (°C); $Q_e$ is the latent energy (MJ/m²·day); $e_s$ is the mean daily vapor pressure at the snow surface (mbar), taken equal to 6.11 mbar (1 mbar = 100 Pa) for a melting snow cover; and $e_2$ is the actual vapor pressure of the air at 2 m (mbar).

Equation [5] has a correlation coefficient of 0.78 and a standard error of estimate of 0.55 MJ/m²·day. Application of the equation should be limited to the months of March and April and to days when $T_{MAX}$ is greater than −5°C. Likewise, exposure, snow cover, and climatological conditions should be similar to those experienced during the 2 years of measurements.

Equation [6] is a modified form of the Kuz'min equation (Kuz'min 1972) used in the USSR for estimating evaporation from a melting snow cover. Usually, during melt of a complete snow cover on the Prairies $Q_e$ is less than 0.065 MJ/m²·day; hence the effects of errors in the term on the overall calculation of melt is small.

Ground-heat flux

The ground-heat flux ($Q_g$) generally falls in the range 0–260 kJ/m²·day. $Q_g$ is assumed small compared with the radiative and convective terms and is neglected in the model.

Heat flux from rainfall

If one assumes that rain falling on a snow cover does not freeze, the energy supplied ($Q_p$) (kJ/m²·day) can be estimated by the expression

$$[7] \quad Q_p = 4.2T_r P_r$$

in which $T_r$ is the temperature of the rain (taken as the mean daily wet-bulb temperature (°C)) and $P_r$ is the daily rainfall (mm).

Changes in internal energy

Shallow snow covers and the upper layers of deep snowpacks often exhibit diurnal cycles in release patterns: e.g., snow melts in later morning and afternoon and refreezes during the night. The nighttime energy deficit must be compensated for the following day before a snow cover releases water. For deep snowpacks the diurnal changes in internal energy are usually small and relatively insignificant in magnitude compared...
with other terms of the energy equation. Therefore, they are frequently neglected in melt calculations. However, such is not the case for shallow snow covers in which changes in internal energy have a direct effect on the energy available for melt and the time of meltwater release.

Internal energy consists of components for the solid, liquid, and vapor phase of the snow and can be evaluated by the expression

\[ u = d\left(\rho_s C_p + \rho_l C_p + \rho_v C_p\right)T_m \]

where \( u \) is the internal energy (kJ/m²); \( d \) is the depth of snow (m); \( \rho \) is the density (kg/m³); \( C_p \) is the specific heat (kJ/kg °C); \( T_m \) is the mean snow temperature (°C); and \( i \), \( l \), and \( v \) refer to ice, liquid, and vapor. Usually the contribution of the vapor phase to \( u \) is negligible. Also, during nonmelt periods the liquid volume can be assumed zero, so that \( u \) can be estimated from measurements of \( d \), snow density, and \( T_m \) (\( C_p \) values can be obtained from handbooks). For days with melt the relative amounts of water and ice in a snow cover may vary widely between day and night. Direct monitoring of internal-energy changes would require detailed, systematic measurements of water and ice contents. At the present time, the only field methods for measuring these variables are dilution techniques, and these are unsuitable for operational practice.

The lack of direct measurements of snow-cover density, phase composition, and temperature required that an algorithm be used to describe \( du/dt \) (11). The routine developed assumes a minimum state of internal energy determined by the minimum daily air temperature; a maximum state equal to zero; a maximum liquid-water-holding content of the snow cover equal to 5% by weight; a snow-cover density of 250 kg/m³; and no melt unless indicated by the albedo subroutine. Complete details on the manner changes in internal energy are accounted for by the system are given in the Appendix.

**Model evaluation**

Several tests were conducted with the EBSM to evaluate its performance in predicting snowmelt and snow-cover runoff and in synthesizing streamflow.

**Start of snowmelt and streamflow**

The ability of the model to predict the “start” of snowmelt and streamflow was studied using meteorological and snow-course data from the Bad Lake Climatological Station and streamflow records from the Creighton Tributary collected in a 13 year period (1972–1984 inclusive). Creighton Tributary is a small watershed (drainage area ≈ 11.4 km²) located about 3 km northwest of the climatological station. The general topography of the catchment is undulating to gently rolling, with approximately 85% of the area under cultivation of cereal grains by dryland farming. It has a well-developed drainage system and does not contain large elements of depressional storage; hence the response time to runoff inputs is relatively short. Streamflow is monitored by a V-notch weir equipped with a stage recorder. Operation of the station is seasonal; that is, it is closed during the winter. Because of the mode of operation, it is possible in some years that the “starting date” of streamflow could be in error (likely less than 2 or 3 days) because of problems in getting the gauge operational.

Figure 3 shows the Julian days of the “start” of snow-cover depletion, determined from changes in the mean depth of snow cover of a snow course located adjacent to the climatological station, and streamflow plotted with the Julian day of “simulated” snow-cover runoff. The data are for years in which the snowfall water equivalent ranged from 32 to 166 mm and include both continuous and interrupted melt sequences. As demonstrated in the figure the agreement of data is good: the correlation coefficient between each measured parameter and simulated runoff is 0.98. As expected, there is a general trend for snow-cover runoff to lag snow-cover depletion (average 1.5 days) and for streamflow to lag snow-cover runoff (average 0.5 days).

To study the performance of the model at a geographic location different from where it was developed and on watersheds with flat terrain and poor relief similar tests were completed using observations from the Regina airport and the Davin Creek and Wascana Creek watersheds near Sedley, Saskatchewan. The distance between the Bad Lake Climatological Station and Regina is about 320 km (southeast), and the hydrometric stations on Davin Creek and Wascana Creek lie approximately 23 km east and 50 km southeast of the airport, respectively. Both Davin Creek (drainage area, 10 km²) and Wascana Creek at Sedley (effective drainage area 350 km²) are subbasins of the Wascana Creek watershed, which drains a large lacustine area in the subhumid region of the province. Both watersheds have well-defined main drainageways; however, Wascana Creek contains large elements of depressional storage, and the area of the basin contributing to streamflow can vary widely from year to year. Approximately 85% of each basin is under cultivation of cereal grains.

The findings of the investigation are summarized below:

1. The agreement between the “start” of snow-cover depletion and snow-cover runoff is excellent (correlation coefficient, 0.99). This result suggests that the empirical relationships for calculating the energy fluxes may be applied to a relatively large area of the central plains of Canada.

2. Large differences were found between the “start” of observed flow and simulated snow-cover runoff, with values ranging from -5 to 17 days on Davin Creek and from -1 to 22 days on Wascana Creek (see Table 1). It should be noted that the differences in start of observed flows from the watersheds are equally large. Although the differences can be attributed mainly to melt patterns, basin-drainage characteristics, and other factors, the possibility that the climatological data at the Regina airport did not provide good representation of conditions on the watersheds must also be considered. For example, in years with above-normal snowfall, the average ratio between snowfall and snow-cover water equivalents at the Regina airport and Davin was 0.68. A proportional decrease in the snow water equivalent in the simulations would advance the “start” of simulated runoff by 1 day on average. Inspection of the data in Table 1 indicates the largest “lags” tend to occur in years with below-normal snowfall. For example, in 1977, 1978, and 1981 observed streamflow lagged snow-cover runoff by 12, 10 and 17 days on Davin Creek and by 22, 16, and 18 days on Wascana Creek. That streamflow often occurs later than direct runoff is attributed to redistribution of the snow cover by wind. In low-snow years the major accumulations are found in local depressions, in drainage-ways, and to the lee of vegetative and topographical barriers. Snow-filled channels store large amounts of water and retard movement. For example, Gray et al. (1985) cited the results of a 1966 snow survey near Regina that showed 12,322 m³ of water per 1000 m of channel when the snow cover had virtually disappeared from adjacent fields and no significant
streamflow had occurred. Field observations have confirmed that significant streamflow does not occur until a channel(s) develops through the accumulations. Lateral inflow of direct runoff to drainageways hastens channelization by supplying energy for melt and by eroding and transporting snow crystals. Because there is a lack of direct runoff to assist the erosional process and because in-channel snow is shaded from the solar beam, melt and the release of meltwater are delayed.

Research is needed toward the development of an algorithm to describe the movement and storage of water in snow-filled channels. These effects were accounted for in the modelling scheme by lagging the snow-cover runoff quantities a fixed time.

Streamflow simulations

The EBSM was interfaced with the U.S. Army Corps of Engineers SSARR model and was used to synthesize streamflow from snowmelt on the Creighton Creek and Wascana Creek watersheds. A few of the simulations, selected for their representativeness, are discussed below. In these comparisons attention should be given to the following: (i) the routing coefficients for SSARR were established from calibration tests with the original system in which melt rates were generated by a temperature-index method (Gray and Landine 1987a; Gray et al. 1986), and they will differ with the EBSM; (ii) no attempt was made to position the simulated hydrographs to obtain “best-fit” agreement with the observed; and (iii) the simulations are for years with near- or above-normal snowfall. The performance evaluation focusses on the agreement in the time elements and shape of the hydrographs.

Simulations were carried out for the Creighton watershed for 1974 and 1975. These two years provided contrasting snow-cover conditions. The winter of 1973—1974 was a year of near-record snowfall that produced an average snow cover of 565 mm having a water content of 143 mm, whereas in 1975 snow-cover conditions were normal with a depth and water equivalent of 299 and 71 mm, respectively. Numerous hydrological studies were conducted on the watershed during these years, and the data sets are considered of unusually high quality. Figures 4 and 5 show the simulated and observed hydrographs. SSARR designates the hydrograph generated by the unrevised model (a temperature-index model is used to calculate melt rates), and EBSM designates the hydrograph produced by the system with the EBSM routine. In 1974, both EBSM and SSARR reproduced the observed hydrograph reasonably well (Fig. 4). The respective values for “efficiency,” \( R^2 \) (Nash and Sutcliffe 1970), were 0.65 and 0.58. That the positive value of \( R^2 \) given by the EBSM is closer to unity suggests a slightly better simulation. With respect to the time elements, SSARR predicted the start of streamflow, whereas the EBSM estimated flow occurring 1 day after the observed. Both models correctly predicted the day of peak discharge.

In 1975 the EBSM gave the closer simulation of observed discharge with an \( R^2 \) value of 0.90, compared with -1.28 given by SSARR. The reason for the better performance was
that the EBSM predicted the start of streamflow within a day of measured flow, whereas runoff from SSARR started 5 days late. In the EBSM surface melt began April 13 because of positive fluxes of net radiation and a daily maximum air temperature $>0^\circ C$. Melt in SSARR did not begin until April 16, after the daily mean temperature exceeded the threshold of $1^\circ C$.

Figure 5 illustrates a weakness in the present version of the ESBM, namely, the inability to properly account for the disposition of snowmelt produced by intermittent melt–freeze events. These often occur on the Prairies in late winter and early spring. Between March 17 and March 23, the ESBM simulated runoff, and these quantities were accepted by the routing routine as contributing to streamflow. A review of radiation records confirmed positive fluxes and melt during this time; however, the maximum daily ambient air temperatures in the same period and the days immediately preceding and following were below $0^\circ C$. Likely, the movement of meltwater runoff would be retarded by the snow cover, and these quantities would not appear as streamflow. Rather, they would infiltrate and freeze in the soil or, following a drop in temperature, refreeze in the snow cover. Allowing meltwater to be released, when in fact it is retained, leads to an underestimation in the snow water available for runoff at a later date.

When surface-melt quantities percolate through the snowpack and infiltrate the soil they change the infiltration potential. Granger et al. (1984) and Gray et al. (1986) showed that the amount of snowmelt infiltration to frozen prairie soils is inversely related to the soil moisture – ice content of the soil layer, $0-300$ mm at the time of melt. Refreezing of infiltrating meltwater decreases infiltration, and a soil becomes impermeable if an ice lens forms on or near its surface. Clearly, an algorithm that would store and refreeze quantities of melt in excess of the liquid-holding capacity of the snow cover and adjust the infiltration potential of a frozen soil would improve both the physical integrity and performance of the EBSM.

In a manner similar to the above, the data in Fig. 5 demonstrate the inability of the degree–day method to account for radiation melt when the ambient air temperature is below freezing.

Figure 6 shows the hydrographs simulated by EBSM and SSARR on Wascana Creek at Sedley in 1974. Note that the lag between snow-cover runoff and streamflow is taken as 3 days, the average for years with normal or above-normal snowfall. The respective $R^2$ values of $-0.05$ and $0.37$ by the models indicate poor agreement with the observed. It can be noted, however, that the EBSM predicted the start of streamflow. The low efficiency is the result of the slope of the rising limb of the simulated hydrograph being much flatter than the observed. This led to a difference in the time of peak flow of 6 days. The slope could be adjusted by changing the routing coefficients, which as mentioned earlier were obtained from calibration.
tests with SSARR. Reasons for the poor simulation by SSARR were due to melt occurring 5 days before measured flow and a sharp decrease in melt rates between April 15 and April 17.

The results of the streamflow simulations with the EBSM on the watersheds over 7 years of record demonstrate that the model is workable and when interfaced with SSARR will provide information on streamflow of accuracy at least equivalent to that obtained with the original system. Also, there is strong evidence that the EBSM will provide better estimates of the time of occurrence of snowmelt and runoff, quantities of melt, and rates of discharge than can be obtained with a temperature-index model. Other advantages of the EBSM over a temperature-index model are (i) it uses the energy equation as the physical framework; (ii) the effects of changes in slope, latitude, atmospheric variables, albedo, and other factors affecting snowmelt can be calculated directly, not compensated for by an adjustment to or recalibration of a single melt factor; and (iii) improved procedures for evaluating different energy terms can be incorporated directly into the system as they are developed. A disadvantage is it requires a larger data base, namely, measurements of sunshine hours, relative humidity, wind speed, and maximum and minimum air temperature. However, at many network stations these observations are routine or they can be measured with relatively simple instruments.

The fact that the EBSM gives reasonable estimates of snowcover depletion and streamflow at locations approximately 320 km southeast of Bad Lake, where it was developed, suggests the empirical relationships that have been presented may
be successfully applied to snowmelt problems over relatively large areas of the Prairies.

**Summary**

An energy-budget model for estimating snowmelt and snow cover runoff is developed and tested. The model is designed for application in operational systems used to forecast streamflow from snowmelt in open prairie environments. It is based on 13 years of field observations collected on the Bad Lake watershed, which is located in the semi-arid, grassland region of central Saskatchewan.

Empirical procedures are presented for estimating radiative, convective, advective, and internal-energy terms from the solar constant and atmospheric transmissivity and daily measurements of snowfall and snow-cover depth; maximum, minimum, and dew-point air temperature, 10 m wind speed, and sunshine hours. An algorithm that accounts for changes in internal energy of a snow cover is described.

The "starting" date of seasonal snowcover runoff simulated by the model is compared with observations of the beginning of snow-cover ablation and the initiation of streamflow from small and large watersheds in the semi-arid and subhumid regions of Saskatchewan. Correlation coefficients between the Julian day of start of snow-cover runoff and snow-cover deple-

be successfully applied to snowmelt problems over relatively large areas of the Prairies.

**Acknowledgments**

The writers wish to acknowledge the financial support provided the project by the Natural Sciences and Engineering Research Council of Canada; the Water Research Support Pro-

gram, Inland Waters Directorate, Environment Canada; and the Research Management Division, Alberta Department of the Environment.

**References**


LIT, R. J. 1968. Smithsonian meteorological tables. 6th ed. The Smithsonian Institution, Washington, DC.


Appendix: Internal-energy algorithm

In developing the algorithm for changes in internal energy two assumptions were made to simplify the accounting procedure. It was assumed a snow cover reached a minimum state if internal energy \( u_{\text{min}} \) was determined by the daily minimum air temperature \( \text{TMIN} (^\circ \text{C}) \); \( u_{\text{min}} \) (MJ/m\(^2\)·day) is calculated by the expression

\[
A1 \quad u_{\text{min}} = 2.5d(2.115 + 0.00779\text{TMIN})/1000
\]

in which \( d \) is the depth of snow cover (cm). The expression in the parentheses \((2.115 + 0.00779\text{TMIN})\) gives the specific heat of ice (kJ/kg·°C) (Dorsey 1940). Equation \([A1]\) assumes a snow-cover density of 250 kg/m\(^3\)—an average value for a nonmelting prairie snow cover. Note: when \( \text{TMIN} = 0^\circ \text{C} \), \( u_{\text{min}} = 0 \). This condition is true for the ice phase, but the liquid content (if any at 0°C) will have an internal-energy content \( u_l \) equal to

\[
A2 \quad u_l = \rho_1 V_l h_l / 1000
\]

in which \( \rho_1 \) is the density of water; \( V_l \) is the volume of water; and \( h_l \) is the latent heat of fusion. However, to simplify the program logic, the maximum value for internal energy was set to zero. The internal energy of the liquid water becomes important only when melt stops. If this occurs, the internal energy of the snow cover is held at zero until its liquid water content is refrozen.

The second assumption is that no melt occurs unless indicated by the albedo routine, i.e., by a rapid decrease in albedo (see Fig. 2).

Figures \( A1a \) and \( A1b \) show a flow diagram of the algorithm. \( Q \) is sum of the major energy fluxes for the day, neglecting \( dV/dt; u_t \) and \( u_{t-1} = \) are the internal-energy contents per unit area on day \( t \) and day \( t - 1 \), respectively \( (u_t \) is the sum of \( u_{t-1} \), the internal energy the preceding day, plus \( Q \), to a maximum value equal to zero and a minimum value of \( u_{\text{min}} \); \( d \) and \( d_{\text{max}} \) are the current snow depth and the maximum snow depth (cm), respectively; \( \text{LW}_{\text{max}} \) is the maximum liquid water capacity of the snowpack; \( \text{ALBEDO DECAY} \) is a logical variable that becomes true when the albedo routine indicates that the albedo is decreasing (melt); \( \text{MELT} \) is the daily amount of snow (ice) converted to liquid; \( \text{DIS} \) is the amount of meltwater released by the snowcover; and \( \text{REF} \) is the portion of liquid water storage that can be refrozen if melt stops.

A brief description of the routine follows. \( Q \), the energy available, is calculated and used to update the status of the internal energy, \( u_t \) (Fig. \( A1a \)). The snowcover depth is checked; if it has increased, \( d_{\text{max}} \) is updated and used to calculate the maximum liquid water content of the snowpack \( (\text{LW}_{\text{max}}) \), which is taken as 5% by weight.

The minimum internal energy is calculated by \([A1]\), and the current internal-energy status \( u_t \) is compared with \( u_{\text{min}} \); if \( u_t \) is less than \( u_{\text{min}} \), it is set to \( u_{\text{min}} \).

If the albedo routine does not indicate melt, or \( u_t < 0 \), then \( \text{MELT} \) and \( \text{DIS} \) are set to 0. Also, if \( u_t < 0 \), a portion of the liquid water in the snowpack is refrozen, the liquid water content is reduced accordingly, and the snow-cover depth \( (d_{\text{max}}) \) is increased by the amount refrozen. Conversely, if \( u_t > 0 \), \( u_t \) is set to zero and control is returned to the main program (excess energy is dumped).

If melt occurs and \( u_t > 0 \) (see Fig. \( A1b \)), then \( \text{MELT} \) (mm/day) is calculated by the expression

\[
A3 \quad \text{MELT} = (u_t/316.8)1000
\]

which is \([2]\) with \( M = \text{MELT} \). \( Q_m = u_t, \rho = 1000 \text{ kg/m}^3, h_l = 333.5 \text{ kJ/kg}, \text{and } B = 0.95 \). If the amount of melt plus the existing liquid water content exceeds the maximum liquid water content, discharge (release of meltwater) begins. Then \( u_t \) is set to zero, and the snow depth is decreased by the amount of melt. When the depth has decreased to zero, the liquid water content is added to the amount of meltwater released in that day.
Fig. A1. (a) Flow diagram for internal-energy accounting.
FIG. A1 (concluded). (b) Flow diagram for internal-energy accounting.