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D. H. Male and Don M. Gray

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Problems in Developing a Physically Based Snowmelt Model

D. H. MALE and DON M. GRAY
Division of Hydrology, College of Engineering, University of Saskatchewan, Saskatoon, Saskatchewan S7N 0W0

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Over the past few years several snowmelt simulation models have been developed as an aid to streamflow forecasting in mountainous regions. This paper describes the major difficulties encountered when simulation of Prairie snowmelt conditions is attempted, not only for the purpose of forecasting streamflow, but also soil moisture, evaporation, and snow distribution patterns. Simulation is discussed in terms of the energy equation for the snowpack and it is shown that the improvement of the model depends on the following factors: (i) the adjustment of the radiation flux at the snow surface for slope and aspect, (ii) the development of procedures which will allow estimates of the areal distribution of sensible heat, (iii) successful modelling of the diurnal freeze-thaw cycle common to Prairie snowpacks, (iv) an investigation of the energy exchange processes during the period when the snow cover is discontinuous or patchy, (v) knowledge of the coupling of heat and mass transfer processes in frozen soils, and (vi) the extrapolation of point estimates of significant parameters to an areal basis.

Introduction

Knowledge of the time and amount of water released from a melting snowpack are important factors which influence the applicability and accuracy of any river forecasting technique or procedure. This is true for both short-term forecasts (1 day to 1 week) which are concerned primarily with flood flows and long-term forecasts which attempt to predict the seasonal water yield. In recent years several snowmelt models have been developed (Anderson and Crawford 1964; Amoroch and Espildora 1966; Anderson and Rockwood 1970; Eggleston, Israelson, and Riley 1971) in an attempt to provide a phenomenological base from which management decisions on river flows may be made. Most of these models have been developed for mountainous, forested basins having deep snowpacks. Their applicability for predicting discharge rates from Prairie watersheds may be questioned simply on consideration of the differences in topographic, climatic, and land use factors between the two regions.

In the semiarid Prairie regions snow constitutes a major water resource. In many locations domestic supplies of potable water originate from snowmelt runoff. In addition, agricultural production depends, to a large extent, on the amount of water which infiltrates the soil during the melt period thus providing soil moisture necessary for germination and the early growth of a crop. Thus, to be of maximum use, a Prairie snowbelt model should be "gener-

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One page of a document in English, discussing snowmelt modeling.

It is much more important to simulate accurately all of the natural processes involved in snowmelt for a 'generalized' model as opposed to a model with only a single output parameter. In a generalized model it is necessary to consider such factors as snow accumulation as it relates to topography and land use, the liquid water content of melting snow, areal variations in snow density, evaporation rates, and underlying ground conditions in more detail than is normally required for the prediction of streamflow.

The major problems encountered in the development of a Prairie snowmelt model are discussed in this paper. Most of these problems arise from the desire to produce a model which can be transposed to similar topographic and climatic regions as the Prairies. A successful model should be applicable to different physiographic regions and not simply 'calibrated' for one set of local conditions.

**The Energy Equation**

A detailed flow chart of a snowmelt simulation model is necessarily extremely complex and difficult to describe in a reasonable space since it includes subroutines to handle such factors as variations in snow cover depth and density, liquid water retention and transmission, routing of the runoff, precipitation in the form of either rain or snow, energy storage in the snowpack, infiltration characteristics, and other variables. However, central to every phenomenological or physically based model is the energy equation. A clearer understanding of the problems associated with model development can be obtained with reference to this equation rather than the details of the computer program.

Consider the energy equation for a snowpack for rain-free periods in the following form:

\[ \frac{dU}{dt} = Q_N + Q_H + Q_L + Q_M \]

where \( \frac{dU}{dt} \) = the rate of change of the internal energy of the snow per unit area (W/m²), \( Q_N \) = the net radiation flux at the snow-air interface, \( Q_H \) = the flux of sensible heat (convection) at the snow-air interface, \( Q_L \) = the flux of latent heat at the snow-air interface, \( Q_M \) = the heat flux at the snow-ground interface, and \( Q_M \) = the flux associated with meltwater leaving the bottom of the snowpack.

Note: the heat flux by rain has been excluded from Eq. [1] as at northern latitudes in the prairie regions of Canada the occurrence of rain during the melt period is highly unusual or an event of very low probability. The manner in which each term is measured or calculated is given in the Appendix.

Equation [1] is written in a form which applies to the total pack. In other words, the snow is considered as a control volume in which the sum of the fluxes at the upper and lower surfaces must be balanced by the rate of change of energy in the entire snowpack. Such an approach is necessary for the relatively shallow snow cover on the Prairies which seldom exceeds 0.5 m in depth in an open field. Conversely for a mountainous snowpack which may be several metres deep it is customary to write the energy equation for the upper surface only and to consider heat conduction and vapor movement from the upper surface to the interior of the pack.

The evaluation of an energy budget model requires that considerable experimental data be accumulated on each of the terms in the energy equation. It is important that each term be measured independently so that an estimate can be made of the accumulated error inherent in the measuring procedures. The resultant error of the budget calculation errors can be considerably larger than any of the individual energy fluxes, depending on climatic conditions, local terrain features, time of day (sun angle), etc. Figures 1 and 2 are typical plots of the terms in Eq. [1] over a 24 h period in the spring of 1974 when melt first occurred. On this day the snow depth was approximately 0.3 m.

These data were obtained from a micrometeorological station located on the Bad Lake Research Watershed in Saskatchewan. Instrumentation at this site included a snow lysimeter which continuously monitors melt and evaporation, 3 twelve level profiles of flux plates and
Fig. 1. Energy fluxes for April 9, 1974, Bad Lake Watershed.

Fig. 2. Comparison of energy fluxes with changes in internal energy, April 9, 1974, Bad Lake Watershed.
Fig. 3. Energy fluxes for March 31, 1974, Bad Lake Watershed.

Fig. 4. Comparison of energy fluxes with changes in internal energy, March 31, 1974, Bad Lake Watershed.
resistance thermometers in the ground to provide estimates of the ground heat flux, a twin probe gamma radiation profiling gauge to measure water movement in the ground, and seven level profiles of wind, temperature, and dew point temperature from which estimates of evaporation, condensation, and sensible heat can be made. In addition, net all wave radiation is measured using a Funk pyrradiometer and incoming and reflected shortwave radiation is monitored with Kipp and Zonen pyranometers. Figures 3 and 4 are similar curves for a day in which no melt was produced. An inspection of these figures shows that the fluxes at the snow-air surface $Q_N$, $Q_H$, and $Q_L$ vary in an irregular manner over a 24 h period. Such variations are the rule except on days with clear skies and steady winds. A simulation of the melt process must consider the influence of these variations, particularly in short term forecasts where they can be very important.

In the following sections each term in the energy budget (Eq. [11]) is considered in some detail in an attempt to show the major difficulties which must be overcome in the development of a model.

Net Radiation, $Q_N$

The term $Q_N$ of Eq. [1] represents the net all-wave radiation flux to or from the snowpack. Its magnitude is affected by the shortwave radiation, consisting of direct solar and atmospheric diffuse components and the net longwave radiation exchange. On the Prairies it has been found that the net radiation flux is extremely important and dominates the thermal regime of the pack during nonmelt periods and also the melt phenomenon when the snowcover is continuous. During the latter part of the melt sequence when the snow cover is patchy the sensible heat flux (convection) becomes equally important and perhaps the dominant factor.

Incident solar radiation and the atmospheric diffuse radiation, both shortwave and longwave, are reasonably uniform over large areas as long as significant changes in atmospheric transparency conditions such as that caused by cloud cover do not occur over the area. Thus point radiation measurements may provide reasonable spatial estimates of the radiative terms over reasonably flat topography.

The reflected shortwave radiation from the snow surface, $Q_m$, is normally calculated from measurements of the incident shortwave radiation, $Q_s$, using the surface albedo $A$, where

$$A = Q_m/Q_s$$

The albedo depends on atmospheric and surface conditions and the properties of the snow. Manz (1974) presented quantitative evidence showing that the albedo is greatly affected by the presence of foreign matter and that the albedo decreased rapidly with an increase in the density and the particle size of the snow.

Figure 5 shows the temporal decay curves of spatial albedo (within the wavelength band 0.2 $\mu$m to 1.2 $\mu$m) measured during 1974 over a lake surface and open fields in Saskatchewan. The curves show three distinct characteristics.

1. High albedos during the nonmelt periods, 70 to 75%.
2. A rapid decay of the albedo of both surfaces during the melt period. Melt on the land surfaces started on April 8.
3. A close association between the albedo of the two surfaces. Major differences between the two curves occur only during the melt period. The more rapid decrease in the lake albedo is likely caused by ponded surface melt and the shallower depth of snow cover on the lake surface. Water began ponding on the lake surface on April 18. By April 24 the land was mostly free of snow and water covered the ice on the lake surface. The result accentuates the importance of snow cover in governing the albedo of a surface.

The albedo decay curves for a Prairie snowpack differ appreciably from those observed for deep mountainous packs. Figure 6 shows the decay in spatial albedo plotted with the age of the melting snow surface (days after the beginning of melt) for Prairie snowpacks and the curve for deep packs used by the U.S. Corps of Engineers (1956). The albedo of the Prairie snowpack exhibits a more rapid rate of decrease with time as the snow cover becomes patchy and the depth of snow decreases, whereby the underlying surface influences the albedo.

O'Neill and Gray (1972) found that, for the Prairies, point measurements of albedo are representative of basin average values during periods of continuous snow cover and to a limited extent during patchy conditions. Thus,
Fig. 5. Comparison of temporal change in spatial albedo over land and ice surfaces, February–April 1974.

Fig. 6. Comparison of the spatial decay of albedo with age of snow for prairie and mountainous snowpacks.

it is possible to use point albedo measurements over relatively large areas (greater than 250 km²), an important consideration when modelling the radiation exchange.

The longwave components of $Q_N$ can be estimated by semiempirical techniques (Brunt 1944; Hoinkes and Untersteiner 1952; Myers 1966), which require the ground surface temperature and an air temperature and humidity measurement approximately 1 m above the ground surface. Unfortunately, it has been shown that the net longwave radiation flux
TABLE 1. Net radiation estimates with direct beam shortwave adjusted for slope and aspect on a small watershed (Bad Lake, March, 1972), units of energy are MJ/m²

<table>
<thead>
<tr>
<th>Watershed units</th>
<th>No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope (°)</td>
<td></td>
<td>4.1</td>
<td>1.8</td>
<td>1.8</td>
<td>20</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>Aspect (°)</td>
<td></td>
<td>101</td>
<td>140</td>
<td>60</td>
<td>189</td>
<td>189</td>
<td>99</td>
</tr>
<tr>
<td>Area (km²)</td>
<td></td>
<td>0.88</td>
<td>0.32</td>
<td>0.36</td>
<td>0.12</td>
<td>0.12</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Date and time
16/1700–17/1700 CST * 1.48 1.77 1.55 –0.38 3.57 1.67
17/1700–18/1700 CST  2.59 2.88 2.65 0.71 4.56 2.78
18/1700–19/1700 CST  0.40 0.40 0.40 0.40 0.40 0.40
19/1700–20/1700 CST  2.58 2.76 2.65 1.40 3.77 2.72
20/1700–21/1700 CST  5.27 5.67 5.00 2.70 8.11 5.54
21/1700–22/1700 CST  2.96 3.39 3.06 0.31 5.74 3.27
Total             45.61 16.87 15.31 5.14 26.15 16.38

*Cloud cover persisted over the watershed.

depends on the vertical distribution of temperature and humidity in the atmosphere (Kon- dredayev 1969) and is not a unique function of air temperature and water vapor pressure near the earth’s surface. Large errors can occur in this term when it is estimated by semiempirical techniques. This is an important limitation in any model since, over extended periods of time, the net longwave flux may be equal to or greater than the net shortwave flux. More elaborate procedures are available but they require detailed measurements of air temperature and humidity variations which are not practical to obtain for the usual simulation studies.

Despite the reservations mentioned above, point radiation measurements are usually adequate to determine the radiation input to a flat snow-covered area. However, problems arise when point measurements have to be adjusted for differences in topography. Table 1 summarizes the results of calculations of the energy flux on a small watershed in Saskatchewan. The watershed was divided into six units each having distinct aspect and topographic features. Hence, the real topographic variations of the basin were represented by six plane surfaces for which representative values of slope and aspect were determined. These values were used to adjust the direct beam component of point shortwave measurements. In Table 1 note the large differences in the calculated energy input for the north-facing (unit 4) and south-facing (unit 5) slopes of the watershed. For the period of calculation the total heat balances on the two units were 5.14 MJ/m² and 26.15 MJ/m², respectively. Also, the daily values of heat input to these two steeply sloping units differ appreciably from the amounts calculated for the other parts of the watershed which have gradual slopes. This feature clearly demonstrates the necessity of adjusting the net radiation term for slope and aspect. While it is possible, if one knows the slope, aspect, and latitude, to calculate the direct solar radiation received by a surface of any orientation the computations required for large regions, even on the relatively flat Prairies, are prohibitive. As yet, no relatively simple operational method has been developed which allows adjustments of slope and aspect to be made as a matter of routine. Similarly, it is possible in principle using the established concepts of the shape factor and emissivity (emittance) to extrapolate point measurements of the longwave radiation flux to a large area. Once again, such calculations are prohibitive and operational procedures must be established for this purpose.

**Sensible Heat, \( Q_H \)**

Under Prairie conditions this flux can also be important in the melt process. It is not uncommon for the sensible heat flux to reach 50% of the maximum net radiation flux on a given day even with wind speeds below 0.5 m/s. As Fig. 1 and 3 illustrate there is not normally a marked diurnal variation in this flux. In an independent evaluation of this term it is necessary to measure the vertical profiles of wind
and temperature above the snow surface. Calculation of \( Q_H \) can then be made using aerodynamic or profile methods which are based more or less rigorously on turbulent boundary layer theory. On the basis of energy balance studies made at the Bad Lake Watershed over the past 3 years it would appear that a form of the Thornthwaite–Holzman equation (1939) gives the most satisfactory estimate of sensible heat over terrain where it is possible to measure a velocity profile which is nearly logarithmic with height.

The use of an equation of this type in a complex snowmelt model is rarely justified because of the large number of temperature and velocity measurements which are required. For example, in the study conducted at the Bad Lake Experimental Watershed seven levels of wind and temperature are measured continuously in the first 2 m of the atmosphere in order to establish the necessary profiles. An approach requiring much simpler input data has been widely used in snow hydrology (U.S. Corps of Engineers 1956; Gold and Williams 1960; Kuzmin 1966) where an equation of the following form is used to determine \( Q_H \):

\[
Q_H = f(U) (T_a - T_s)
\]

where \( f(U) \) = an empirically determined wind function, \( T_a \) = the air temperature, and \( T_s \) = the temperature at the snow surface.

The accuracy of such expressions is difficult to determine and depends largely on the wind function \( f(U) \) which must be evaluated for each local condition. From the energy balance studies conducted on the system it can be inferred that it is possible to calculate \( Q_H \) with an accuracy of from 10 to 20% although errors of 50% or greater are possible at wind speeds below 0.5 m/s.

The values given above restrict the accuracy of any snowmelt model but probably a much more serious limitation is the applicability to field conditions of expressions such as Eq. [3]. Fundamental to all such expressions is the assumption that the sensible heat flux, \( Q_H \), is constant with height in the surface boundary layer. It is essential to the accurate determination of \( Q_H \) that such a boundary layer exist and that it be of a thickness that will allow measurement of the temperature and velocity profiles. Meaningful measurements of these profiles are not possible unless the local terrain is relatively flat and free from obstructions. This restriction makes it impossible to estimate the sensible heat flux in many natural situations. Furthermore, even on relatively flat terrain, such as on the Prairies, rolling hills, clumps of trees, river banks, and road-cuts make it extremely difficult to relate a point measurement of the sensible heat flux to any representative areal value. Thus it is necessary to search for some index station where the 'mean areal wind' and the 'mean areal temperature' are assumed to exist. While such a procedure is feasible over a relatively small watershed having a more or less uniform terrain (Carlson, Norton, and Britch 1972) the limitations imposed by our current knowledge of the areal variation in the sensible heat flux is the major obstacle in the development of a snowmelt model which can be transposed from one area to another.

The sensible heat flux is also difficult to evaluate when the snow cover is patchy; that is, when the snow has melted from fallow fields, hill tops, and other areas where the depth is a minimum. Gray and O’Neill (1974) have shown that significant amounts of heat are advected from snow-free areas and used to melt the snow on adjacent snow-covered areas. In the latter part of the melt period this transfer dominates the thermal regime of the pack. It is during this period that streamflow increases significantly. The different aerodynamic formulas are difficult to apply in this situation where the boundary layer characteristics are continuously changing. Furthermore, little information is available on the variations in the sensible heat flux over a patch of snow and on the adjacent bare ground. An instrument has been developed which is designed to measure \( Q_H \) directly (Bailey, Mitchell, and Beckman 1973) and which can be used in areas where well developed boundary layer profiles are not present. Systematic measurements from such an instrument possibly would provide the information necessary to give reasonable areal estimates of \( Q_H \). Unfortunately the instrument has yet to be evaluated under cold weather conditions and some modifications undoubtedly will be necessary before it can be used in the field to provide data necessary for operational applications.
Evaporation, $Q_E$

The magnitude of this term is directly related to the vapor pressure gradient above the snow surface and the wind velocity. Studies conducted at the Bad Lake Watershed show that during nonmelt periods the net amount of evaporation/condensation over a 24 h period is generally negligible (see Fig. 3). Although under winter conditions significant evaporation can occur in mid-day (near solar noon), this flux is usually balanced by an equal amount of condensation in the early evening. As melt progresses the net evaporation, although small, is measurable (0.1 mm/day) but when the pack reaches a depth of less than 5 cm evaporation rates of 0.3 mm/h have been measured.

Calculation of evaporation rates (like sensible heat rates) are normally made using aerodynamic formulas. Comparative tests have been made to determine the applicability of the different formulas used to calculate evaporation rates based on data collection on the lysimeter. The results suggest that the equation developed by Thornthwaite and Holzman (1939) is best suited to Prairie conditions. Figure 7 gives a comparison between calculations of evaporation based on this equation and the lysimeter measurements for 2 days in March 1973. The agreement of 'calculated' and 'measured' quantities is satisfactory considering that semie empirical equations were used and the experimental error. Use of this equation does require the simultaneous measurement of the wind and humidity profiles above the snow surface. Nevertheless, with the current state of knowledge, for modelling purposes it is advisable to use equations of a form analogous to Eq. [3] as the required input data is much less.

The limitations of the different formulas used for calculating sensible heat apply equally and perhaps more so to the calculation of evaporation rates, because of the difficulties inherent in measuring vapor pressure gradients. There are no established procedures for determining if point measurements of evaporation are representative of areal conditions. Given the low rates of evaporation measured it is unlikely that evaluation of the evaporation/condensation term will be a serious limitation in any model as long as the snow cover is continuous.

The lysimeter studies show that once the snow cover becomes patchy evaporation rates can increase significantly. Presumably evaporation takes place at an accelerated rate from the edges of the pack and at other points where the snow depth is small so that the underlying surface becomes an important factor in the energy exchange. In separate studies on isolated snowpatches both Rechard and Raffelson...
(1974) and Gray and O'Neill (1974) conclude that 20 to 30% of the water equivalent volume of the snowpatch evaporates or sublimes. On the Prairies, deep snowpatches frequently remain and contribute runoff for 1 to 5 weeks after fallow and stubble fields are bare. Thus the loss of water through evaporation can be a significant factor in the prediction of seasonal water yield. Physically based methods for calculating the long-term evaporation during this period are not available. However, efforts are being directed to develop empirical relationships which would apply to these conditions.

**Ground Heat Flux, Q_G**

Accurate simulation of the heat flux across the soil-snow interface is complicated by the effects of many properties and processes, for example, the soil moisture content, infiltration of melt, water vapor transfer, and the magnitude of the solar radiation penetration through the pack. This latter factor becomes significant when the snow cover is shallow (2–4 cm). Under such conditions the temperature of the snowpack may be increased significantly because of the interaction of the interface on the radiative transfer processes. The transfer of heat from the ground to the snow is one of the factors which contributes to the accelerated rate of evaporation at the edge of a snowpatch and to the melt of shallow packs.

For deep snowpacks, values of Q_G are relatively small, which suggests that a simple average value may be used with a reasonable degree of confidence. The U.S. Corps of Engineers (1956) suggest that a nominal energy flux of approximately 2 W/m² be attributed to heat supplied by the soil to the snowpack. Gold (1957) measured an average heat flux of 10 W/m² from the soil to the pack at Ottawa. Yoshida (1962) observed melt rates equivalent to 3–4 W/m² at the snow-ground interface of a deep pack in Japan. Measurements with heat flux plates, confirmed by corresponding measurements of temperature and thermal conductivity, give values in the range 0–3 W/m² for Prairie conditions. These values are insignificant in most energy balance considerations. Because of its small magnitude this flux is not included in Figs. 1 and 3 although the measured fluxes are included in the summation term of Figs. 2 and 4.

For simulation purposes Q_G can be safely ignored in short-term forecasts (1 week or less); however, since it does not normally change sign over a 24 h period its cumulative effect can be significant for seasonal forecasts and hence should be included in long-term studies.

The flux of heat from the ground to the snowpack is also important as it affects the temperature regime of the underlying soil and hence the infiltration rate at the time of melt. Under certain conditions the infiltration rate of a soil may be increased when frozen because of increases in the size and volume of voids caused by structural changes which accompany the freezing process. Conversely, under other conditions if the soil temperature is below 0 °C, infiltrating water may be refrozen in the surface layers resulting in the formation of an impervious layer which restricts infiltration and increases the runoff potential.

The physics of soil moisture movement under partially or completely frozen conditions is not fully understood. This problem, in part, may be attributed to the lack of development of instruments which will provide accurate measurements of important physical parameters of the soil which govern its water transmission properties. Unquestionably, the infiltration process in a frozen soil is a complicated thermodynamic problem which can only be solved by coupling the amount of heat transferred by mass transport (infiltrating water) with that transferred by conduction, convection, and radiation processes.

**Internal Energy of the Snowpack (dU/dt)**

As shown in Figs. 2 and 4 it is evident that significant changes in this term are extremely important in energy balance studies of the relatively shallow snowpacks which are common to the Prairies. Experimental results have shown that this term is extremely difficult to measure with a high degree of accuracy and confidence. The difference between dU/dt and the sum of the energy fluxes apparent in Figs. 2 and 4 represents the magnitude of the error which may be expected when efforts are made to measure all components of the energy balance. Most of the error can be attributed to the internal energy term. Fortunately, for most
FIG. 8. Simulation of snowpack temperatures on the lysimeter prior to melt using air and ground temperatures as input.

TABLE 2. Snowpack temperature correlation matrix (showing correlation coefficients between air and ground temperatures and the temperatures at different depths within the snowpack)

<table>
<thead>
<tr>
<th>Top snowpack (−1 cm)</th>
<th>Middle snowpack (−6 cm)</th>
<th>Bottom snowpack (−13 cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air temperature*</td>
<td>0.987</td>
<td>0.884</td>
</tr>
<tr>
<td>Ground temperature†</td>
<td>0.629</td>
<td>0.917</td>
</tr>
</tbody>
</table>

*Air temperature taken 20 cm above the ground.
†Ground temperature taken 4.8 cm below ground.

operational purposes it is usually not necessary to make predictions on an hourly basis and the errors involved are considerably reduced when balances over a 24 h period or longer are computed. On days when no melt occurs cumulative errors of less than 10% are not uncommon.

The internal energy term consists of a component for the solid, liquid, and vapor phases of the snow and has the form

\[ U = \sum_{i} L \rho_i C_{vi} + \rho_l C_{vl} + \rho_v C_{v} \]  

where \( L \) = the snow depth, \( \rho \) = the density (mass per unit volume of snow), \( C_v \) = the heat capacity, and \( T_m \) = the mean snow temperature. The subscript \( i \) refers to ice, \( l \) to liquid, and \( v \) to vapor.

During nonmelt days for all practical purposes the liquid density can be assumed to be zero and the simulation of the internal energy changes involves the estimation of snow depth, mean temperature, and snow density. Dybvig (1974) found that during these periods the temperature regime within a shallow snowpack and hence the internal energy changes could be predicted using a simple conduction model based on the solution of the unsteady state diffusion equation. Figure 8 shows a typical set of results obtained with the model which demonstrates that the predicted and measured temperatures are in sufficiently close agreement for simulation purposes. Dybvig tested the sensitivity of the model to different inputs, namely, net radiation, ground heat flux, and air and ground temperatures. His findings indicated little improvement in the predictability of the model from using flux values over results obtained using temperature data. Table 2 gives the correlation matrix between air, ground, and snow temperatures for nonmelt days. The correlation coefficients indicate that the variation in the air and ground temperatures explain 97.4% and 93.7% of the variation in the temperatures at the top and bottom of the pack.

On days when liquid water is present in the pack the situation is considerably more complicated. On these days it is necessary to stimulate the changes in \( \rho_l \). This is important during the initiation of melt. During this period on the Prairies runoff is commonly produced in the afternoon but the pack cools at night and
freezes and is subject to large changes in internal energy content. The following day a significant amount of the net incoming energy is required to bring the pack to the point where melt water is again produced. Unless this process can be modelled successfully predictions of the amount of melt water released on any given day can be seriously in error. Current snowmelt models treat only the liquid water variations in a snowpack at 0°C (Anderson 1972). This is a serious limitation in areas of shallow snow cover since the isothermal condition within the pack exists only part of the day. Attempts are being made to develop a model for the Prairies which will overcome this problem. The procedure being evaluated involves the use of an analysis developed by Colbeck (1974) in which the movement of water through snow is assumed to occur under the influence of gravity and to obey Darcy's Law. While Colbeck's analysis is limited to an isothermal pack at 0°C it has been found that a numerical solution involving this analysis and the principle of energy conservation shows promise of being able to model the freeze–thaw cycle adequately. A comparison of the results of this analysis with snow lysimeter measurements taken in the spring of 1974 show that the time at which measured and predicted melt begins agree within 15 min over a 3 day period in which the freeze–thaw cycle was present. Further testing of this component is necessary to determine its sensitivity to various sets of boundary conditions.

A further complication involving the internal energy term arises when the area distribution of the snow is considered. On the Prairies, snow depth varies widely depending on land use and vegetal, topographic, and climatic conditions. Runoff does not occur simultaneously from the areas. It has been found that runoff amounts can be overestimated in the order of several magnitudes unless the major source areas are identified (Gray and O'Neill 1974). Thus, it is necessary to model the temperature regime of the snow and water movement through the pack on a spatial basis. Melt is produced first from the shallow depths of snow and often the snow has completely disappeared from these areas before the deeper packs begin to produce. Simulation of this phenomenon depends on an accurate classification of the topographical features of an area according to their snow retention characteristics. Suitable classifications are being developed (Steppuhn and Dyck 1974) but they have not yet been incorporated into a snowmelt model. Experience has shown that accurate estimates of snow depth and density probably are the initial prerequisites to development of a snowmelt model based on the energy budget concept.

Finally, any model must be supplied with data periodically so that forecasts may be updated. This is particularly critical for the internal energy term since it has a direct bearing on the melt produced. Updating of this term involves, among other measurements, a systematic measurement of the liquid water content of the snowpack. The only field methods available to date for this purpose consist of cumbersome calorimeter techniques which are not suitable for operational practice. Of the many remote sensing techniques currently being developed only the microwave techniques (Meier and Edgerton 1971; Linsor, Meier and Smith 1974) have the capability of collecting the necessary information. These techniques show promise because they will enable both the liquid water and the ice contents of the snow to be determined.

Conclusion

A paper of this nature which concentrates on problems encountered in the development of a snowmelt model necessarily gives a distorted view of the usefulness of such devices as a management tool. It must be emphasized that several of the existing models have produced good results in deep mountainous snow packs in regard to both the reproduction of major snow cover variables and the reproduction of discharge hydrographs. Once the required information is available models for Prairie regions will be equally successful (Anderson 1972). Current and future research aimed at overcoming the present deficiencies in simulation models should be directed to:

1. The development of an operational methodology for adjusting the net radiative flux for slope and aspect which will contain procedures to be followed in partitioning a watershed as a simple terrain model.

2. The development of techniques whereby...
the sensible and evaporative heat fluxes at the snow–air interface may be evaluated on a spatial basis. Given the current state of the art in boundary layer theory these procedures will necessarily be empirical or semiempirical.

(3) The further development of the techniques for simulating the water movement through snow including the associated problem of the lag and storage effects of snowpacks on the runoff sequence.

(4) The development of procedures by which the major source areas of runoff on a spatial scale can be identified. This will necessarily require accurate spatial estimates of snow water equivalent and pattern of snow cover disappearance.

(5) The development of methodologies and techniques of partitioning the energy fluxes to the different components under partial or patchy snow cover conditions.

(6) Studies of the factors affecting the infiltrations of water to frozen soils.

(7) The development of empirical relationships through which different energy fluxes may be evaluated from network hydrometeorological data.

Hopefully, these studies will lead to the development of forecast models which can be used as an aid in resource management and not simply increase the complexity of existing models. In this way they will become valuable aids for data requirement and network design problems.


MALE AND GRAY: SNOWMELT MODEL

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Appendix I. Evaluation of Energy Budget Components

Figures 1–4 show the energy fluxes measured at a point on the Bad Lake Research Watershed in Saskatchewan during 2 days in 1974. This Appendix briefly outlines the methods and techniques used to evaluate each of the terms in Eq. [1]. For convenience Eq. [1] is reproduced below:

\[ dU/dt = Q_X + Q_H + Q_K + Q_N + Q_M \]

where \( dU/dt \) = the rate of change of the internal energy of the snow per unit area (W/m²), \( Q_X \) = the net radiation flux at the snow-air interface, \( Q_H \) = the flux of sensible heat (convection) at the snow-air interface, \( Q_L \) = the flux of latent heat at the snow-air interface, \( Q_K \) = the heat flux at the snow-ground interface, and \( Q_M \) = the flux associated with meltwater leaving the bottom of the snowpack.

It is important to note that the terms on the right hand side of the equation are fluxes; that is, they represent rates of change of energy across the snow-air or snow-ground interface. The rate of change of internal energy \( (dU/dt) \) is not a flux in that there is no exchange mechanism associated with it. Hence the different notation. The method used to evaluate each term in Eq. [1] is outlined below.

\( Q_X \), Net Radiation

This flux is measured using a Funk net pyranometer located directly over the snow lysimeter. Readings from this instrument were compared with a net radiometer located approximately 80 m from the lysimeter. Agreement between the two instruments was found to be generally good.

\( Q_H \), Sensible Heat

A form of the Thornthwaite–Holzman equation is used to estimate this term,

\[ Q_H = C_p k^2 U_1(T_2 - T_1) / \left[ \ln \left( \frac{z_1}{z_0} \right) \ln \left( \frac{z_2}{z_1} \right) \right] \]

where \( C_p \) = specific heat of air (kJ/kg °C), \( k \) = Von Karman constant, \( U_1 \) = wind velocity at 20 cm (m/s), \( T_2 \) = temperature at 20 cm (°C), \( T_1 \) = temperature at 20 cm (°C), \( z_0 \) = roughness height determined in neutral conditions (cm), and \( z_1 \) and \( z_2 \) refer to the heights 20 cm and 63.3 cm respectively.

Data for use in this equation are obtained from a mast located near the lysimeter on which seven levels of air temperature, wind, and dew point temperature are obtained continuously during the measurement period.

\( Q_L \), Evaporation and Condensation

This term is obtained directly from lysimeter measurements. Evaporation is detected as a change in weight by three compression load cells. The precision of the evaporation measurement in terms of an equivalent depth is ±0.02 mm.

Evaporation is also estimated from profile measurements of wind and humidity using the Thornthwaite–Holzman equation. As discussed in the body of the paper the agreement between these two methods is generally good.

\( Q_K \), Ground Heat Flux

This term is measured by heat flux plates installed at three locations in the soil immediately adjacent to the lysimeter. Each installation includes 12 flux plates located at depths ranging from 0.1 cm to 2 m. In addition, ground temperature measurements are made at each flux plate location and used to calculate the heat flux from the Fourier heat conduction equation. To date the two methods of obtaining this term agree within 10% of each other.

\( Q_M \), Melt

The lysimeter is capable of measuring this term directly. Melt water is collected in a tank which is weighed continuously using a tension load cell. In addition, the amount of water is
measured by means of a pressure transducer located at the bottom of the tank and by measuring the height of water manually. Very close agreement among the three methods has been achieved to date.

\( dU/dt \), Internal Energy

An examination of Eq. [4] reveals that in order to evaluate this term it is necessary to measure the snow depth, snow density, the liquid water content of the snow, and the average snow temperature.

Each of these measurements is made as follows: depth, obtained manually over the lysimeter by means of a metre stick; temperature, obtained from resistance thermometers inserted in the snow at approximately 5 cm depth increments; density, obtained from known volume of snow on lysimeter and the weight of snow; and liquid water content, obtained using a calorimeter (Yosida 1960).

It should be noted that four separate measurements are involved in the evaluation of this term. Hence it is not surprising that cumulative errors, particularly on days of rapid melt can be as high as 40%.