Steady-state suspension of snow

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ABSTRACT


A procedure to calculate the steady-state mass concentration, mass flux and transport rate of suspended blowing snow over continuous snowcovers has been coupled to extensive measurements of the mass flux of suspended snow. Theoretical modelling of the vertical diffusion of snow particles undergoing sublimation specifies that the suspended snow mass concentration is controlled by a dimensionless vertical velocity and the height of a reference concentration near the lower boundary of suspended flow. Whilst the result for short time scales is affected by variable boundary conditions (surface erosion, dune formation, snowfall), on a storm-event time scale only height, wind speed and surface roughness need be considered to calculate mass concentration. The development of an expression for mass flux and integrating it through the appropriate boundary layer provides a suspension transport model that may be implemented with standard meteorological service wind-speed measurements. The model indicates the transport rate of suspended snow increases approximately with the fourth power of the 10 m wind speed. As wind speed increases, suspended transport rates rapidly exceed those of saltating transport; suspension comprises over 90\% of total snow transport for wind speeds in excess of 17 m s\(^{-1}\). Application of the suspended blowing snow model using meteorological data will assist in determining snow redistribution and the over-winter hydrological balance of wind-swept, snow-covered catchments.

INTRODUCTION

Redistribution of snow by wind transport can significantly effect the depth, density and distribution of the resulting snowcover with important implications for surface energy exchange, snowmelt dynamics and annual water balances. Modelling snow redistribution in a physically based manner promotes an understanding of the underlying transport processes their interaction with various landscape and meteorological regimes and the potential for management.

Wind transport of snow primarily occurs in two modes, suspension and
Saltation. Saltation is the dominant mode within a narrow layer of the atmosphere near the surface, whilst suspension dominates at heights above this layer. This discussion is concerned with the suspended component of blowing snow and with modelling its behaviour over uniform terrain. It is shown that modelling the turbulent diffusion of blowing snow from first principles is not possible given the present understanding of the phenomenon. However, a dimensionless parameter is defined from first principles and used to interpret measurements of the mass flux of suspended blowing snow over continuous snowcovers and to provide a means of calculating the transport rate of suspended blowing snow from standard meteorological measurements.

Suspended blowing snow consists of snow particles supported by atmospheric turbulence that follow erratic downwind paths which do not regularly intercept the snow surface. The source of these particles may be entrained surficial snow or snowfall. Paths of suspended particles have a vertical component which results from a balance between gravitational acceleration and a drag force exerted by the fluctuating vertical wind speed. Downwind transport occurs as the mass of particles moves with the mean horizontal component of the wind speed.

Suspended blowing snow particles are subject to sublimation during transport (Schmidt, 1972) and abrasion if they are derived from the saltation layer; hence they have usually been observed as metamorphosed spheroids of ice rather than crystal forms (Budd et al., 1966; Schmidt, 1982). These observations lead to the assumption that their density is approximated by that of ice. Pomeroy and Male (1988) used this assumption and the assumption that the suspended ice particles are spherical in shape to calculate visual range in blowing snow from the mass concentration and particle size. The calculations compare well with the visual range, particle size and mass concentration measurements of Budd et al. (1966) in the Antarctic, suggesting that the particle density assumption is appropriate.

Budd (1966) and Schmidt (1982) present measurements of suspended snow particle size. Budd (1966) fitted the distribution of particle radii to a two-parameter gamma distribution of the form

\[ f(p) = \frac{p^{(a-1)} \exp\left[-(p/\beta)^a\right]}{\beta^a \Gamma(a)} \]

where \( f(\ ) \) denotes the frequency, \( p_r(m) \) is the snow particle radius, \( a \) is a shape parameter, \( \beta(m) \) is a scale parameter and \( \Gamma \) is the gamma function. The mean radius of suspended particles is equal to \((a/\beta)\). Budd found the mean particle radius to decrease from near 100 \( \mu \)m near the surface to 40 \( \mu \)m at a height of 2m and \( a \) to increase from 5 to greater than 15 for these same heights. In the lowest metre of the atmosphere, Schmidt (1982) found an
exponential decrease in snow particle radius with increasing height and a proportional increase in $\alpha$ with height. The range of suspended snow particle radii is from 10 to 170 $\mu$m (Budd, 1966; Schmidt, 1982).

Upward diffusion of blowing snow occurs from a source near the snow surface. Budd et al. (1966) suggested the source height was just above the snow surface and equivalent to the aerodynamic roughness height; the order of 1 mm. A balance of atmospheric lift and drag forces exerted on the cohesive snowpack surface led Schmidt (1980) to conclude that these forces are insufficient to directly entrain snow particles at the surface. Schmidt suggested that the impact force exerted by saltating particles is more likely to entrain surface snow. Saltation is a bounding motion along the surface in which saltating particles eject surface particles upon their collision with the surface. Saltation makes surface snow amenable to turbulent diffusion by ejecting particles to several centimetres above the surface and by abrading surface snow crystals to smaller sizes (Pomeroy and Gray, 1990). Pomeroy and Male (1987) presumed diffusion of blowing snow proceeds from within the saltation layer and calculated a coefficient of transfer from saltation to suspension based upon the probability of the upward turbulent velocities exceeding snow particle fall velocities at a height of 10 mm above the surface. However, a more likely source of suspended snow particles is the upper saltation layer, where particle trajectories are notably disturbed by turbulence (Hunt and Nalpanis, 1985; Anderson, 1987). As recognized by Radok (1968) and others (Schmidt, 1986; Pomeroy and Male, 1987) the height at which particles become suspended in this manner is affected by the size and trajectories of saltating particles and the characteristics of atmospheric turbulence near the snow surface.

Shiotani and Arai (1953) noted the decline in mass concentration (mass of blowing snow per unit volume of atmosphere) with distance from the surface. Based upon the assumption that the diffusivity of snow equals that of momentum within the atmospheric boundary layer, they used diffusion theory to develop an expression relating the mass concentration $\eta(\text{kg m}^{-3})$ at some height $z(\text{m})$ to a reference mass concentration at $z_1$ near the surface where

$$\eta(z) = \eta(z_1) \left( \frac{z}{z_1} \right)^{-[\alpha/(k u^*)]}$$

The friction velocity, $u^*(\text{m s}^{-1})$, is equal to $(\tau/\rho)^{0.5}$ where $\tau(\text{N m}^{-2})$ is the atmospheric shear stress and $\rho(\text{kg m}^{-3})$ is atmospheric density. Von Kármán's constant, $k$ is taken as 0.4 and the terminal particle fall velocity, $\omega(\text{m s}^{-1})$ is assumed constant with height. Mellor and Radok (1960) used this form to describe mass concentration profiles from the Antarctic. Budd (1966) provided a better fit with Antarctic mass concentration profiles by modifying
the equation to account for the gamma distribution of snow particle sizes and the variance of this distribution with height. Schmidt's (1982) measurements did not agree with Budd's model, in that the exponent in eqn. (2) was not well described by estimated fall velocities and friction velocity.

A second source of suspended snow particles is falling snow, which in field conditions is usually difficult to distinguish from eroded surface snow. When wind speeds are sufficiently high to support the vertical diffusion of surface snow, falling snow particles are subject to significant turbulent velocity fluctuations that reduce deposition velocities and modify particle size and shape through sublimation. Particle sizes and gradients of mass concentration are modified by falling snow, though analyses of this effect in fully developed boundary layers are lacking (Schmidt, 1984; Pomeroy, 1991). An upper boundary to the blowing snow boundary layer during snowfall is suggested, by the practice in industrial aerosol studies, as the height where the blowing snow mass concentration is within 1% of the mass concentration of snow due to precipitation alone.

Observations in Antarctica by Budd et al. (1966), suggest that suspended blowing snow without snowfall, may extend, in conditions of virtually unlimited snowcovered fetch and high wind speeds, to heights of 300 m. Dyunin and Kotlyakov (1980) suggest that in limited-fetch environments, sublimation of snow particles prevents diffusion of snow in significant quantities to heights above a few tens of centimetres. In regions of limited uniform fetch, the near-surface blowing snow boundary layer must lie within the momentum boundary layer, whose growth is predictable (Pasquill, 1974), however snow may extend above this layer. In stable atmospheres, the height of a strong inversion forms an upper boundary for blowing snow.

MODEL CONCEPT

The purpose of the suspended blowing snow model is to calculate the downwind transport rate of snow as controlled by various boundary-layer parameters. To be useful in examining the snow hydrology of various climates and terrains, the suspended transport model should use synoptic meteorological data and interface with available models for saltation, snow surface momentum exchange (Pomeroy and Gray, 1990) and sublimation (Schmidt, 1972). Whilst limited to steady-state conditions, the model should provide an initial step towards an unsteady flow model.

A control volume of unit area, extending from the saltation/suspension boundary to the top of the fully developed blowing snow boundary layer is the format for the model of suspended snow. The model assumes for the control volume a steady state, in that a uniform fetch of continuous
Control Volume for Suspended Snow Transport

snowcover prevails for a suitable upwind distance and that an atmospheric boundary layer has developed over this fetch. Figure 1 shows the control volume with inputs of inflowing suspended snow, entrained saltating snow and falling snow and outputs of outflowing suspended snow, resettling suspended snow and sublimating snow.

THEORETICAL CONSIDERATIONS

Transport rate

The transport rate of suspended blowing snow is the integral of the mass flux between the height of the lower boundary for suspension, at \( z = h^* \) (the saltation/suspension interface), and the height of the upper boundary, at \( z = b \). Hence the suspended snow transport rate, \( Q \), \((\text{kg} \, \text{m}^{-1} \, \text{s}^{-1})\) as mass of snow per unit width per unit time is,

\[
Q = \int_{h^*}^{b} q(z) \, dz
\]  

(3)

The mass flux of suspended snow at height \( z \), \( q(z) \), \((\text{kg} \, \text{m}^{-2} \, \text{s}^{-1})\) is the product of the mean horizontal particle speed, \( u_p \, (\text{m} \, \text{s}^{-1}) \) and the mass concentration at \( z \), \( \eta(z) \) where

\[
q(z) = u_p(z) \eta(z)
\]

(4)

Schmidt (1982) presents optoelectronic measurements of horizontal snow particle velocity and mean wind speed at heights from 50 mm to 1.0 m. Whilst
there are residuals between the two measurements, the mean difference is small and there is no systematic deviation between the two values with increasing height. This suggests that under steady-state conditions and for heights greater than 50 mm, suspended snow particles have achieved the horizontal wind speed. In the constant shear stress or ‘fully developed’ atmospheric boundary layer where momentum turbulence dominates, the wind-speed gradient is approximated as a logarithmic form,

\[
    u(z) = \frac{u^*}{k} \ln \left( \frac{z}{z_0} \right)
\]  

(5)

The aerodynamic roughness height, \( z_0 \) is the hypothetical height above the surface of the zero wind speed (Morris, 1989). Schmidt (1982), Pomeroy and Male (1987), Pomeroy and Gray (1990), Tabler et al. (1990b) and Pomeroy (1991) concluded that this equation provides sufficient fit to wind-speed gradients measured in blowing snow under a variety of atmospheric temperature and humidity gradients when \( z_0 \) is permitted to vary with friction velocity.

Approximating the mean particle speed with the wind speed at the height of the particle yields the mass flux relationship,

\[
    q(z) = \eta(z) \left( \frac{u^*}{k} \right) \ln \left( \frac{z}{z_0} \right)
\]  

(6)

Based upon this relationship and Budd et al.’s (1966) estimated mass concentrations, Greeley and Iversen (1985) suggest that the mass flux of suspended blowing snow increases as the cube of the wind speed.

Combining eqns. (3) and (6) provides the transport rate of suspended snow, \( Q \) as a function of the wind speed and mass concentration,

\[
    Q = \frac{u^*}{k} \int_{z_0}^{z} \eta(z) \ln \left( \frac{z}{z_0} \right) \, dz
\]  

(7)

Turbulent diffusion theory must be applied to develop a method to calculate \( \eta(z) \).

**Turbulent diffusion**

Turbulent diffusion theory is developed in analogy to the diffusion of a gas contaminant in a fluid. This application is to particles of appreciable terminal fall velocities in a turbulent gas. The change in mass concentration with time \( d\eta/dt \) in some differential element \( dx \times dy \times dz \times dt \) of time and space
(t, x, y, z) may be defined as (with all parameters referenced to t, x, y, z)
\[ \frac{d\eta}{dt} = -\mu_p \frac{\partial \eta}{\partial x} - w \frac{\partial \eta}{\partial z} - v_{y(p)} \frac{\partial \eta}{\partial y} - \frac{\partial \eta}{\partial t} \] (8)

In eqn. (8) \( \frac{\partial \eta}{\partial t} \) represents a mass transformation such as sublimation, \( w (\text{m s}^{-1}) \) is the vertical particle velocity and \( v_{y(p)} (\text{m s}^{-1}) \) is the y-direction particle velocity.

The mass flux in a particular direction may be envisioned as composed of a fluctuating and a mean component. For example, the vertical mass flux \( q_v \) is described by the expression
\[ q_v = (w\eta) + (w\eta)' \] (9)
where \((w\eta)\) is the mean vertical flux and \((w\eta)'\) is the deviation from the mean due to atmospheric turbulence. The fluctuating component of eqn. (9), averaged over time, is equal to the turbulent diffusivity of the snow particles, \( \kappa_s (\text{m}^2 \text{s}^{-1}) \) multiplied by the mass concentration gradient in analogy to Fick’s law, where for blowing snow,
\[ (w\eta)' = -\kappa_s(z) \frac{\partial \eta}{\partial z} \] (10)

Using eqns. (9) and (10), the mass fluxes in the x-, y- and z-directions to an elemental volume per unit time can be calculated. The sum of the mass fluxes is the change in mass \( d\eta/dt \) within the volume.

On a large, flat, uniform surface, the atmospheric boundary layer becomes fully developed in that downwind momentum gradients in the x-direction are negligible. This condition exists within the lowest few metres of the atmosphere over much of the snowcovered prairies, arctic tundra and continental ice-sheets of the high latitudes. Applied to blowing snow mass concentration, \( \frac{\partial \eta}{\partial x} = 0 \) in the fully developed boundary layer. In extremely stable conditions the wind direction is not constant with height, however, on average, aligning the coordinate system along the mean wind direction negates fluxes in the cross-wind or y-direction, hence \( \frac{\partial \eta}{\partial y} = 0 \). For blowing snow the only mass transformation is sublimation, hence the diffusion equation reduces to a one-dimensional vertical gradient, segregated into mean and fluctuating components, and sublimation:
\[ \frac{d\eta}{dt} = -\frac{\partial [w\eta - \kappa_s(\partial \eta/\partial z)]}{\partial z} - V_s \eta \] (11)
where \( V_s \) is a highly variable rate coefficient (s\(^{-1}\)) which accounts for sublimation of snow particles (positive for conversion from vapour to solid) and \( V_s \eta (\text{kg} \text{m}^{-3} \text{s}^{-1}) \) is a rate of sublimation per unit volume of atmosphere (usually zero or negative).
For steady-state conditions, \( \frac{d\eta}{dt} = 0 \). This condition is approximately satisfied where fetch is adequate to allow a fully developed atmospheric boundary layer over \( z \), the extent of the ‘cloud’ of suspended blowing snow particles lies within the range of \( z \) and values are time-averaged to eliminate the turbulent component.

A solution to eqn. (11) may be found by setting boundary conditions, examining the variation in \( w \) with height, estimating \( V_s \) and calculating \( \kappa_s \). The following discussion addresses the solution for \( w \) and \( V_s \) when gradients of the mass concentration with height are known.

**Boundary conditions**

The lower boundary (see Fig. 1) corresponds to the saltation–suspension ‘interfaces’, an artifact that simplifies the modelling of blowing snow. The flow of snow near the interface has a high mass concentration and involves a mixture of saltating and suspended particles. Hunt and Nalpanis (1985) and Pomeroy (1988) suggest that saltating particles at this height have trajectories modified by turbulence. The boundary is set at \( z = h^* \), where \( h^* \) is ‘near’ the mean trajectory height of saltating particles. At height \( h^* \), all saltating particles are undergoing trajectories modified by turbulence and provide a source for suspended particles. To provide continuity and to couple the calculation of saltation and suspension transport, \( h^* \) is the height where mass concentrations begin to decline with increasing height from their mean saltation value. The mean saltation mass concentration is found from Pomeroy and Gray’s (1990) formulation as

\[
\eta_{\text{salt}} = \frac{\rho}{3.29u^*} \left( 1 - \frac{u_n^{*2}}{u^*2} - \frac{u_t^{*2}}{u^*2} \right)
\]

where \( \rho \) is the atmospheric density, \( u_n^* \) is the friction associated with non-erodible surface roughness elements, and \( u_t^* \) is the friction velocity at the threshold of transport. The boundary conditions at \( h^* \) are:

\[
-w\eta + \kappa_s(z = h^*) \frac{d\eta}{dz} = q_v(z = h^*) = E
\]

where \( E(\text{kg m}^{-2}\text{s}^{-1}) \) is the vertical mass flux in an upward direction from the saltation layer. For a steady-state saltation layer \( E \) is the net erosion rate of snow from the surface. When the downward flux of snow due to snowfall is greater than the upward flux (both at the surface), \( E \) assumes a negative value.

The upper boundary for turbulent transfer of blowing snow is the upper boundary of the atmospheric flow layer that meets the following conditions:

1. constant shear-stress layer, \( \partial\tau/\partial z = 0 \);
(2) steady-state suspension, \( \frac{d\eta}{dt} = 0 \);
(3) evidence of diffusion of snow, not just snowfall, \( \frac{\partial \eta}{\partial z} \neq 0 \);
(4) suspended snow, \( \eta \neq 0 \).

The height of the upper boundary occurs from near the surface to tens of metres above, and is usually controlled by the vertical extent of the 'cloud' of diffusing blowing snow. The vertical extent of this cloud can be limited by the height of an atmospheric inversion layer but usually increases with horizontal distance (fetch) from an obstruction to blowing snow, such as a wooded area. Defining \( z = b \) as the height of the upper boundary,

\[
-w\eta + \kappa_s(z = b) \frac{\partial \eta}{\partial z} = q_s(z = b) = -S
\]

where \( S \) is the rate of snowfall (the vertical mass flux in a downward direction at height \( b \)). Note that for non-snowfall conditions, \( w\eta \) near height \( b \) becomes small, hence

\[
\kappa_s(z = b) \frac{\partial \eta}{\partial z} \approx 0
\]

Conversely, during notable snowfall, \( \partial \eta / \partial z \) becomes small and hence,

\[
-w\eta = -S
\]

For steady-state conditions, erosion at the lower boundary balances snowfall and sublimation. Thus

\[
-S - E - V_r m_{h \rightarrow b} = 0
\]

where \( m_{h \rightarrow b} \) is the mass of blowing snow within a column of unit horizontal area extending vertically from height \( r \) to \( b \).

**Fall velocity**

The non-fluctuating component of particle velocity, \( w \), equals the negative of \( \omega \), the mean terminal fall velocity of particles in still air. For non-uniform particles, the fall velocity of the particle of mean mass is appropriately employed in a mass-flux model. The terminal fall velocity of a particle is a manifestation of the atmospheric drag and gravitational forces acting on a particle. Thus, \( \omega \) represents the mean velocity of the particles relative to the mean vertical velocity of a fluid point. Lee (1975) found Carrier's (1953) drag relationship best reproduced the measurements of Beard and Pruppacher (1969) for water droplet fall velocity, given Reynolds numbers less than 2. Carrier's relationship between particle drag coefficient \( c_D \) and particle
Reynolds number, \( Re \) is

\[
C_d = \frac{24}{Re} (1 + 0.0806 \text{Re}) \tag{18}
\]

Using this relationship, Pomeroy and Male (1986) developed an approximation \((R^2 = 0.99)\) of the solution for the terminal fall velocity of a snow particle as a function of particle radius, \( p_r \), as:

\[
\omega = (1.1 \times 10^7) p_r^{1.8} \tag{19}
\]

Budd (1966) and Schmidt (1981) have shown the distribution of suspended blowing snow particle radii fits a gamma distribution. Schmidt (1982) fitted vertical profiles to measurements of blowing snow mean radius, \( \langle p_r \rangle \), and gamma distribution shape parameter, \( \alpha \). Pomeroy (1988) averaged those profiles \((R^2 > 0.95)\) to develop the following expressions:

\[
\langle p_r \rangle (z) = 4.6 \times 10^{-5} z^{-0.258} \tag{20}
\]

and

\[
\alpha(z) = 4.08 + 12.6z \tag{21}
\]

The mean mass of a blowing snow particle, \( \langle m \rangle (\text{kg}) \) is found as a function of \( \langle p_r \rangle \) and \( \alpha \) following an approximate solution of the gamma distribution equation as

\[
\langle m \rangle = \frac{4}{3} \pi \rho_i \langle p_r \rangle^3 \left( 1 + \frac{3}{\alpha} + \frac{2}{\alpha^2} \right) \tag{22}
\]

where \( \rho_i (\text{kg m}^{-3}) \) is the density of ice. Using eqn. (22) to find the radius of mean mass as a function of mean radius and \( \alpha \) and combining with eqns. (19), (20) and (21) provides an expression for the fall velocity that is approximated by

\[
\omega_{\langle m \rangle} (z) = 0.1910 z^{-0.5395} \tag{23}
\]

Hence from considerations of particle drag and particle size–height relationships in the literature, the mean mass terminal fall velocity of suspended blowing snow particles is found to be inversely related to the square root of height above the surface. This variation with height has important implications for the solution of the diffusion model.

Sublimation, snowfall and erosion

Estimates of \( V_{s, \langle m \rangle} \) can be made using the sublimation rate coefficient and the mass of suspended snow in a column located between heights \( h^* \) and
The mass of suspended snow, $m_{h\rightarrow z}$, increases exponentially with $z$. The sublimation rate coefficient, $V_s$, is a function of atmospheric turbulence, temperature, humidity, and the mean particle size and may be calculated by various schemes (Schmidt, 1982; Pomeroy, 1988) based upon Schmidt's (1972) blowing snow sublimation model. Atmospheric gradients of turbulence velocities, mean particle size, temperature and humidity lead to complex gradients of $V_s$ with height. While $V_s$ is expected to become more negative with height, the rate of change may vary dramatically with a change in the atmospheric environment.

The erosion rate $E$ is equal to $(-V_s m_{h\rightarrow z} - S)$. Pomeroy (1988) estimates values of $V_s m_{h\rightarrow z}$ from 0 to $-7.6 \text{gm}^{-2}\text{s}^{-1}$ for western Canadian climatic conditions. Stallabrass (1987) has measured snowfall rates as great as $S = 3.0 \text{gm}^{-2}\text{s}^{-1}$. The term $(-V_s m_{h\rightarrow z} - S)$ hence should vary from $-3.0$ to $7.0 \text{gm}^{-2}\text{s}^{-1}$ in response to snowfall rate, temperature, humidity and wind speed.

Based on these estimates, the erosion rate $E$ varies within a range from $-3.0$ to $7.0 \text{gm}^{-2}\text{s}^{-1}$ and the term $(E + V_s m_{h\rightarrow z})$ decreases with height from $E$ at the saltation interface to $-S$ at the top of the suspended layer. Its gradient with height is controlled by the snowfall rate, the mass concentration gradient and the atmospheric environmental conditions that influence sublimation.

**Turbulent diffusivity of blowing snow**

The turbulent diffusivity of blowing snow in the atmosphere, $\kappa_s$, is analogous to the viscosity of a fluid. Sommerfeld and Businger (1965) suggested $\kappa_s = 10 \kappa_m$, where $\kappa_m(\text{m}^2\text{s}^{-1})$ is the eddy diffusivity, easily calculated from the vertical profile of wind speed, however, most researchers have assumed $\kappa_s \equiv \kappa_m$. For the present discussion let us assume a linear relationship between the parameters, with the eddy diffusivity defined for conditions of strong mechanical mixing as,

$$\kappa_s(z) = c \kappa_m(z) = cu^*kz$$

where $c$ is a proportionality coefficient.

**Solution for the diffusion model**

The snowfall rate at the top of the atmospheric boundary layer is not normally measured and blowing snow at such heights is of less interest to surface snow hydrology that are the fluxes near the surface. Therefore, the
The diffusion equation is integrated for $\frac{d\eta}{dt} = 0$ with reference to the saltation-suspension interface. For any height, $z$, located between $h^*$ and $b$,

$$\omega_{(m)}(z)\eta(z) + \kappa_s(z) \frac{\partial \eta}{\partial z} = E + V_s(z)m_{h^*\rightarrow z} \tag{25}$$

where $[E + V_s(z)m_{h^*\rightarrow z}]$ is the net vertical flux of suspended snow at height $z$.

Equation (25) must be integrated to find the mass concentration at some height. To prepare it for integration, $(cu^*kz)$ is substituted for $\kappa_s(z)$ and the terms rearranged so that

$$\frac{cku^*}{\eta(z)} \frac{\partial \eta}{\partial z} = \left[\frac{E + V_s(z)m_{h^*\rightarrow z}}{\eta(z)} - \omega_{(m)}(z)\right] \frac{\partial z}{z} \tag{26}$$

The term $[E + V_s(z)m_{h^*\rightarrow z}]\eta(z)(m\ s^{-1})$ is comparable to a vertical velocity.

While the left-hand side of eqn. (26) may be easily integrated, the form of the variation of $[E + V_s(z)m_{h^*\rightarrow z}]\eta(z) - \omega_{(m)}(z)$ with $z$ cannot be specified in general. One approach to a solution is to integrate eqn. (26) over small intervals, $dz$, for which

$$E + V_s(z)m_{h^*\rightarrow z} - \omega_{(m)}(z) = w'(z) \tag{27}$$

and the term, $w'(z)$ is a modified vertical velocity. For any interval $dz$ of $z$ bounded by $h^*$ and $b$ eqn. (27) becomes,

$$cku^* \int_{z}^{z+dz} \frac{1}{n(z)} \partial \eta = w'(z) \int_{z}^{z+dz} \frac{1}{z} \partial z \tag{28}$$

Evaluating the definite integrals from $z$ to $(z + dz)$ gives

$$w'(z) = cku^* \frac{\log \left[\eta(z + dz)/\eta(z)\right]}{\log (z + dz/z)} \tag{29}$$

and defining a dimensionless vertical velocity, $w^*$ as

$$w^*(z) = \frac{w'(z)}{cku^*} \tag{30}$$

eqn. (29) can be solved to find a mass concentration, $\eta(z + dz)$:

$$\eta(z + dz) = \eta(z) \left(\frac{z + dz\ \w^*(z)}{z}\right) \tag{31}$$

As the variation of $w^*$ with height is not known, iterations of eqn. (31) calculate the mass concentration profile over a range of heights from a reference mass concentration. The following section presents an analysis of
TABLE 1

Meteorological and snowpack conditions during blowing snow measurements

<table>
<thead>
<tr>
<th>Day</th>
<th>( u^* ) (m s(^{-1}))</th>
<th>Snowfall (mm snow-water equivalent)</th>
<th>Snow depth after drifting (mm)</th>
<th>Standard deviation of snow depth (mm)</th>
<th>Snow surface hardness after drifting (kN m(^{-2}))</th>
<th>Air temperature (°C)</th>
<th>Relative humidity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18 January</td>
<td>0.2</td>
<td>4.5 on 17 January</td>
<td>98</td>
<td>40</td>
<td>370</td>
<td>+1 to -17</td>
<td>50–60</td>
</tr>
<tr>
<td>20 January</td>
<td>0.27</td>
<td>2.2 concurrent with rain</td>
<td>115</td>
<td>34</td>
<td>625</td>
<td>+1 to -3</td>
<td>50–62</td>
</tr>
<tr>
<td>26 January</td>
<td>0.23–0.27</td>
<td>None</td>
<td>103</td>
<td>28</td>
<td>760</td>
<td>-2 to -7</td>
<td>55–65</td>
</tr>
</tbody>
</table>

measured mass concentrations and their heights in an attempt to develop a functional relationship between \( u^* \) and height, find characteristic reference mass concentrations and hence model the change in \( \eta \) with height.

CONTRIBUTION OF EMPIRICAL OBSERVATIONS

Measurements

For 18, 20 and 26 January 1987, 237 vertical profiles of the flux of suspended blowing snow particles, mean wind speed, air temperature and humidity were determined from 7.5 min averaged measurements over a plain uniform surface of summer fallow (soil cultivated so that all vegetation is removed), overlain by a complete snowcover, extended 600 m upwind of the measurement site. The site lies 4 km west of the urban limits of Saskatoon, Canada, at an elevation of 500 m above sea-level. Conditions during the 3 days of measurement are characterized in Table 1 and may be summarized as:

(1) 18 January: blowing snow after fresh snowfall, rapidly declining temperature;
(2) 20 January: heavy snowfall and blowing snow with freezing rain later in the afternoon;
(3) 26 January: intermittent blowing snow after several days of wind-hardening of the snow surface.

Despite occasional above-freezing temperatures, no snowmelt occurred during these periods because of a net loss of radiation from the snowcover. Snow depths varied on the field from 250 to 50 mm. Mean snow depths and standard deviations after the blowing snow events are shown in Table 1. Snow surface hardness increased notably after snow, blowing snow and freezing
rain on the 20 January, and again after intermittent blowing snow and wind-hardening on the 26 January. Air temperature and humidities are shown in Table 1. Notable is the decline in air temperature from 1 to $-17^\circ$C during measurements on 18 January. Humidities generally increased during a blowing snow event.

Wind speed was measured with 'Qualimetrics' three-cup anemometers at six levels, logarithmically spaced from 0.35 to 3.0 m above the snow surface. Cup rotation was accumulated over 7.5-min periods and converted to mean windspeed using a U.S. National Bureau of Standards calibration. Air temperature was measured using 'YSI thermilinear' thermistors at five levels from 0.1 to 2 m above the snow surface. The thermistors are housed in tubular double-radiation shields orientated into the wind; a fine-mesh (105 $\mu$m mesh) filter fabric prevents blowing snow from entering the tubes. Precision of these thermistors is within 0.1$^\circ$C with a response time of 10 s. Humidity was measured using 'Honeywell' lithium-chloride electro-chemical hygrometers at five levels in tandem with the thermistors. YSI thermilinear thermistors monitor the core temperature of the hygrometer bobbins. The heated bobbins are housed in radiation shields and aspirated using a central air pump. Air intakes to the radiation shields are protected from snow particles using the fine-mesh filter fabric. The precision of the hygrometers is within 0.3$^\circ$C dew point temperature at air temperatures near freezing and declines somewhat with temperatures.

The flux of blowing snow particles was measured with optoelectronic snow particle detectors (Brown and Pomeroy, 1989) located at five levels, spaced logarithmically from 0.01 to 2 m above the snow surface. The particle detector filters out signals from extremely small particles ($< 22 \mu$m radius) hence is not affected by most airborne dust. The particle flux is converted to a mass flux using typical particle size distributions found in blowing snow (Schmidt, 1981) to correct for small, uncounted particles and the variation of the sampling area with particle size. Comparisons of the mass flux calculated from the particle detector output to the mass of snow accumulated in a filter-fabric sediment trap show mean differences of 0.00211 kg m$^{-2}$ s$^{-1}$ for mass fluxes from 0.04 to 0.19 kg m$^{-2}$ s$^{-1}$ (Brown and Pomeroy, 1989).

**Vertical profiles of mass concentration**

*Measured mass concentration*

Measured blowing snow mass flux corresponds to $q(z)$, from which $\eta(z)$ may be calculated using eqn. (6). Measured vertical profiles of wind speed were used to calculate $u^*$ and $z_0$, following eqn. (5). Equation (6) may overestimate particle speeds for heights near the saltation layer (less than
0.04 m above the snow surface) where some particles are in transition from saltation to suspension and have not achieved full atmospheric horizontal velocity. For this reason measurements made at heights near the estimated saltation layer are excluded from the analysis.

Variation with height

Typical measurements of mass concentration profiles are shown in Fig. 2 for a variety of wind speeds on the 3 days of measurement. They are characterized by an exponential decrease in mass concentration with height resulting in values at a 1 m height three to four orders of magnitude less than those near the surface. Measurements at the lowest level, especially on 20 January, are of snow in the transition from saltating to suspended snow; gradients of mass concentration tend to meet at a focus of constant mass concentration at this transition.

The dimensionless vertical velocity, $w^*$ was calculated using eqns. (29) and (30) for segments of the mass concentration profile between measurement levels. In Fig. 3, $w^*$ for each height segment is plotted against time for the 3 days of measurement. Though $w^*$ tends towards a constant value at each mean height, a cyclical variation over time is sometimes apparent, being most pronounced at lower heights. This variation is not associated with a variation in the friction velocity. Changes in $w^*$ at the cessation of snow transport late in the day on 18 and 26 January are also prominent. The variation may be categorized according to measurement- and phenomenon-related effects. One measurement effect is the passage of snow dunes, which changes the measurement height above the snow surface by several centimetres. These changes in measurement height could not be recorded in the field, hence an inaccuracy results in the analysis of $w^*$ with height. The inaccuracy was most prominent during snowfall on 20 January, when partial gauge burial occurred, and in general is most pronounced for values of $w^*$ measured near the surface where a small difference in $z$ can have a tremendous effect on $w^*$.

The diffusion model predicts a variation in $w^*$ with the vertical snow flux. At lower heights, this flux is strongly affected by the surface erosion rate, $E$, which is in turn controlled by the interaction between saltating snow and the snow surface. Some of the cyclical variation in $w^*$ measured at the lowest height may indicate variation in the erosion rate and the saltation process. If, as is widely postulated in the sediment transport literature, saltation and erosion are modified by the passing of dunes, it becomes difficult to distinguish between inaccuracies in the measurement height and fluctuations in the surface erosion rate on the cycling of $w^*$ at heights near the surface.

During snowfall, the vertical snow flux is strongly affected by the snowfall rate. The scatter in $w^*$ during snowfall on 20 January is instructive. According
Fig. 2. Measured vertical profiles of snow mass concentration. Averages over 7.5 min of mass concentration, calculated as mass flux divided by flow velocity.
Fig. 3. Dimensionless vertical velocity time series. Averages over 7.5 min of \( w^* \) calculated from the vertical gradient between two adjacent mass concentrations. Heights shown are geometrical means of the two measurement heights.
to the diffusion model an increased downward snow flux makes \( w^* \) more negative, suggesting that the highly negative values of \( w^* \) on 20 January are associated with 'bursts' of heavy snowfall. Interestingly, measurements at the middle level showed the most stability for all conditions, suggesting that suspended snow near the saltation layer and at heights approaching 1 m above the surface (where mass concentrations are normally low) display greater relative effects from snowfall.

Measurements collected on 18 January, when the air temperature fell sharply over the day from +1 to -17°C, permit an examination of the influence of sublimation on \( w^* \). The fall in air temperature corresponds to an order of magnitude decrease in the sublimation rate given the relatively constant relative humidity of about 60%. Despite the decrease in sublimation rate throughout the day, no trend in \( w^* \) with time is evident for 18 January in Fig. 3. The few highly negative \( w^* \) values measured at the 0.18 m height towards the end of the day are associated with an increased downward flux of snow as the wind speed dropped and drifting ceased. This result suggests that variation in the sublimation rate coefficient may be countered by a complementary variation in the vertical snow flux, i.e. an increase in sublimation is matched by an increase in snow erosion (an attractive hypothesis in terms of a steady-state mass balance).

In Fig. 4 the dimensionless vertical velocity is plotted against the geometric mean of height above the snow surface for the 3 days of measurement. The reduction in \( w^* \) with height is evident, with the most scatter at high levels associated with snowfall on 20 January and at low levels with the passage of snow dunes. Because of the variation induced by snowfall on 20 January, a relationship between the vertical velocity, \( w^* \), and height was determined from
measurements without snowfall, on 18 and 26 January. For the 324 observations of mass concentration and wind speed without snowfall, a least-squares linear regression on log-transformed data suggests the relationship,

\[ w^* = -0.8412z^{-0.544} \]  

(32)

with a coefficient of determination, \( R^2 \), of 0.82. Figure 4 also shows predicted \( w^* \) plotted against height. As is evident in the figure, errors in the estimation balance out over time, the mean difference between the full set of measured and predicted \( w^* \) values being 0.061 over a range from 0.1 to 5.5.

**Reference mass concentration**

Equation (31) requires a reference mass concentration in order to solve for mass concentrations at other heights. By extrapolating measured mass concentrations towards the snow surface, using eqns. (31) and (32), appropriate reference values for the measurements may be calculated. An analysis of snow saltation (Pomeroy and Gray, 1990) suggests that 'mean' mass concentrations in saltation tend to fall between 0.4 and 0.9 kg m\(^{-3}\) irrespective of friction velocity, and that the mean height of the saltation trajectories increases with the square of the friction velocity. Choosing a 'constant' reference mass concentration that would hypothetically always fall within the saltation layer permits the calculation of a variable height at which this reference would hypothetically occur. Setting the reference mass concentration, \( \eta_r \), equal to 0.8 kg m\(^{-3}\), 225 reference heights for diffusion were calculated from measured mass concentration profiles on 18, 20 and 26 January. The heights are shown in Fig. 5, plotted against the friction velocity, with which they show a linear increase. A least-squares linear regression between the reference height for calculation of diffusion (given a mass concentration of 0.8 kg m\(^{-3}\)), \( z(\eta_r = 0.8) \), and friction velocity, \( u^* \), yields the relationship,

\[ z(\eta_r = 0.8) = 0.05628u^* \]  

(33)

with a coefficient of determination, \( R^2 = 0.76 \), and a standard error of estimate equal to 0.0036 m out of reference heights that vary from 0.01 to 0.1 m. Inclusion of measurements collected during snowfall does not substantially alter the fit or slope of this relationship. Note that the saltation–suspension interface height, \( h^* \), forms the actual lower boundary for suspension of snow and 'does not necessarily correspond' to \( z(\eta_r = 0.8) \).

**Calculation of mass concentration**

Equation (31) may be recast as a differential equation and solved using eqns. (32) and (33) for the mass concentration as a function of height and friction velocity. Thus

\[ \eta(z) = 0.8 \exp \left[ -1.55(4.784u^*^{-0.544} - z^{-0.544}) \right] \]  

(34)
where the constant, 0.8, has units of kilograms per metre cubed. Using this procedure, mass concentrations were calculated from wind speeds and heights corresponding to the 761 measured mass concentrations and compared with these measurements. The comparison covers all days of measurements and both snowfall and non-snowfall conditions. Variance between modelled and measured values increases with mass concentration. To correct this heteroscedasticity, logarithms of values are used for comparison and plotted in Fig. 6. The coefficient of determination between modelled and measured values is $R^2 = 0.84$ and the standard error of estimate is 0.0016 kg m$^{-3}$ over
a range from $1 \times 10^{-6}$ to $1 \text{kg m}^{-3}$ of the 761 comparisons. Separation of comparisons into snowfall and non-snowfall conditions indicates that the model predicts mass concentration equally well for both conditions, despite the equation for $w^*$ being derived exclusively from non-snowfall measurements. The degree of prediction indicated by the $R^2$ and standard error of estimate suggest that the central tendency in mass concentration during both snowfall and non-snowfall blowing snow events is adequately specified by the model for time-averaged calculations, though the scatter in Fig. 6 suggests that considerable disparity is possible in comparing single measurements with the model.

IMPLEMENTATION OF THE MODEL

*Friction velocity from wind-speed measurements*

During blowing snow, the aerodynamic drag of saltating particles modifies the wind-speed profiles above the saltation layer, increasing aerodynamic roughness from the non-transport level (Chamberlain, 1983). Pomeroy and Gray (1990) fitted the following expression to wind-speed measurements during blowing snow over a completely snowcovered field:

$$z_0 = 0.1203 \frac{u_*^2}{2g}$$  \hspace{1cm} (35)

The coefficient in eqn. (35) is greater than those measured by Tabler (1980) and Schmidt (1982) and others referenced by Morris (1989), the difference due to less efficient saltation rebound on the ice-free Saskatchewan snowcovers (Pomeroy and Gray, 1990). Combining eqns. (5) and (35) provides the friction velocity in terms of the 10 m wind speed, $u_{10}$, that is commonly measured by meteorological stations:

$$u^* = 0.02264u_{10}^{1.295}$$  \hspace{1cm} (36)

*Saltation-suspension interface*

The height $h^*$ that delineates the saltation and suspension layers and sets the lower boundary condition for the suspension model is found by extrapolating the modelled suspension mass concentrations towards the saltation layer until they approximate the mean saltation mass concentration as calculated using eqn. (12). Figure 7 shows this interface height given saltation mass concentrations calculated for a complete, continuous snowcover and three different threshold friction velocities (representing three different snowcover cohesion/hardness states). At friction velocities above threshold
levels, the difference in threshold friction velocities makes little difference in the interface height. The increase in interface height, $h^*$, with friction velocity may be approximated as

$$h^* = 0.08436u^{*1.27}$$  \hspace{1cm} (37)$$

The interface height suggested by this equation does not increase with friction velocity as sharply as the maximum saltation trajectory height proposed by Owen (1964) (and somewhat differently as referenced by Greeley and Iversen, 1985), though the two values correspond closely to low friction velocities.

**Suspended mass flux and transport rate**

The mass flux of suspended blowing snow was calculated as a function of the 10 m wind speed for a range of heights from the saltation–suspension interface up to 10 m; the results are shown in Fig. 8. Not only the mass fluxes themselves increase with wind speed but the vertical profile of mass flux displays a less dramatic decline with height as wind speed increases. This behaviour suggests a rapid increase in suspended snow transport rate as wind speed increases. Integrating the mass flux from the suspension–saltation interface to an arbitrary height of 5 m provides a measure of the suspended snow transport rate. A boundary-layer height of 5 m or greater develops over a snowcovered, unobstructed upwind fetch of about 500 m (Pomeroy, 1988). The suspended snow transport rate is plotted in Fig. 9 as a function of 10 m wind speed along with the saltation transport rate from the model of Pomeroy and Gray (1990). Suspended transport rapidly exceeds the saltation transport rate as wind speed increases from the threshold condition. The increase in
suspended transport rate with wind speed for $u_{10}$ from 6.5 to 25 m s$^{-1}$ is approximated by,

$$Q_{\text{susp}} = \frac{u_{10}^{4.13}}{674100}$$  \textmd{(38)}

where $Q_{\text{susp}}$ (kg m$^{-1}$ s$^{-1}$) is the suspended transport rate in a column extending from the saltation layer to a 5 m height. The rapid increase in suspended transport with wind speed causes it to dominate total blowing snow transport. Compared with saltation transport as shown in Fig. 9, suspended transport

Fig. 9. Suspended blowing snow transport rate. The modelled transport rate of suspended blowing snow as a function of the 10 m wind speed. The saltation transport rate (Pomeroy and Gray, 1990) is plotted for comparison.
comprises about 75% of total transport at wind speeds of 8 m s\(^{-1}\) and over 90% of total transport for wind speeds greater than 17 m s\(^{-1}\).

Verification of this transport rate is provided by several sources and for several geographical regions. Pomeroy (1989) found the blowing snow transport rate calculated using this suspension model interfaced to a saltation model (Pomeroy and Gray, 1990), the combined model termed the Prairie Blowing Snow Model or PBSM, corresponded well with Schmidt's (1986) height-integrated measurements of snow transport. Tabler et al. (1990b) integrated Mellor and Fellers' (1986) regression analysis of Antarctic suspended snow mass flux profiles, constraining mass fluxes in the saltation layer, to produce an expression with results that compare well with those produced by the PBSM. Tabler et al. (1990a) compared snow transport calculated by an approximation of the PBSM results using measured wind speeds with measurements of snow accumulation behind snow fences over 4 years at Prudhoe Bay, Alaska. Referring to the approximation of the PBSM they concluded that

"when basic requirements for snow conditions and wind data are satisfied, the above relationship provides reasonably close estimates of seasonal snow transport over heights 0–5 m at Prudhoe Bay. No other known formulation relating wind speed to transport provides approximations that agree as well with measured snow accumulation."

They also noted that large errors can accrue from failure to satisfy the requirement for unlimited snow. Pomeroy (1991) demonstrates a procedure to adjust the suspended snow transport rate for losses from vertical transport and sublimation, when the supply of surface or falling snow is limited.

CONCLUSIONS

A procedure to calculate the steady-state mass concentration, mass flux and transport rate of suspended blowing snow over complete and continuous snowcovers has been coupled to measurements of the mass flux of suspended snow. The results help to define a turbulent diffusion model for snow. The model elucidates several features of the steady-state snow diffusion phenomenon over complete snowcovers which include the following.

1. The mass concentration of blowing snow decreases exponentially with height and increases exponentially with atmospheric friction velocity, changing by four to five orders of magnitude over the range of heights and wind speeds found in the lowest metre of the atmosphere.

2. The vertical gradient of suspended snow mass concentration is
controlled by a dimensionless vertical velocity and the height of a reference mass concentration near the saltation-suspension interface.

(3) The reference mass concentration height is insensitive to changes in vertical snow fluxes, sublimation and saltation dynamics and is solely a function of wind speed and surface roughness.

(4) Because of the characteristics of the dimensionless vertical velocity on short time scales (minutes), the vertical gradient of suspended snow mass concentration is sensitive to changes in the vertical flux of snow at the lower boundary (surface erosion, dune formation) and upper boundary (snowfall), however, for longer time scales (storm events or hours) the gradient is insensitive to changes in these fluxes and is solely a function of height, wind speed and surface roughness.

(5) The vertical gradient of suspended snow is insensitive to changes in the sublimation rate at time scales from minutes to hours; even relatively rapid sublimation shows no sign of changing the dimensionless vertical velocity and hence depleting the supply of suspended snow in the lowest 2m of the atmosphere.

(6) The horizontal transport rate of suspended snow increases as approximately the fourth power of wind speed. This rapid increase causes suspension to supersede saltation as the primary blowing snow transport mechanism as wind speed increases from the transport threshold level. Suspension comprises over 90% of total snow transport for 10 m wind speeds in excess of 17 m s$^{-1}$. The understanding of blowing snow diffusion and suspended transport provided by this study permits modelling of transport rates over level, complete snowcovers from standard meteorological data and provides a basis for modelling the phenomenon in more complex flow and boundary conditions.

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