PROPAGATION OF MOISTURE AND TEMPERATURE WAVES INTO FROZEN SOIL

Litong Zhao and Donald M. Gray
Division of Hydrology
University of Saskatchewan
Saskatoon, Saskatchewan
CANADA

ABSTRACT

The results of a numerical study of the propagation of moisture and temperature waves into a homogeneous, unsaturated, frozen soil are presented. The equations describing the transient processes are based on a local volume averaging formulation of transport phenomena in porous media. Solutions were obtained iteratively using an implicit, finite difference method. They are used to study heat and mass transfer with phase changes in a silty clay soil during infiltration. The simulations show that the propagation speed of temperature is always higher than that of moisture, and the depth of penetration of the temperature wave is always deeper than that of moisture. The effects of initial soil conditions of surface saturation, saturation, and temperature on the speed of propagation and the depth of penetration of the waves are discussed.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>heat capacity, J/kg·K</td>
</tr>
<tr>
<td>D</td>
<td>dimensionless ratio, dimensionless penetration depth, Z/D</td>
</tr>
<tr>
<td>g</td>
<td>gravity, m/s²</td>
</tr>
<tr>
<td>h</td>
<td>enthalpy change, J/kg</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity, W/m·K</td>
</tr>
<tr>
<td>K</td>
<td>saturated liquid permeability, m²</td>
</tr>
<tr>
<td>Kₕ</td>
<td>hydraulic conductivity, m/s</td>
</tr>
<tr>
<td>Kᵢ</td>
<td>unsaturated liquid permeability, m²</td>
</tr>
<tr>
<td>L</td>
<td>length scale, m</td>
</tr>
<tr>
<td>m</td>
<td>dimensionless phase change rate</td>
</tr>
<tr>
<td>M</td>
<td>phase change rate, kg/m³·s</td>
</tr>
<tr>
<td>P</td>
<td>dimensionless parameter</td>
</tr>
<tr>
<td>pₑ</td>
<td>dimensionless capillary pressure</td>
</tr>
<tr>
<td>Pₑ</td>
<td>capillary pressure, N/m²</td>
</tr>
<tr>
<td>Pe</td>
<td>Peclet number</td>
</tr>
<tr>
<td>q</td>
<td>dimensionless heat transfer rate</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>S</td>
<td>saturation, = (θ₁ − θ₃) /θ</td>
</tr>
<tr>
<td>t</td>
<td>time, s</td>
</tr>
<tr>
<td>T</td>
<td>temperature, K</td>
</tr>
<tr>
<td>vᵢ</td>
<td>dimensionless liquid velocity</td>
</tr>
<tr>
<td>Vᵢ</td>
<td>liquid velocity, m/s</td>
</tr>
<tr>
<td>z</td>
<td>dimensionless length</td>
</tr>
<tr>
<td>Z</td>
<td>length, m</td>
</tr>
<tr>
<td>β</td>
<td>empirical constant</td>
</tr>
<tr>
<td>φ</td>
<td>dimensionless temperature</td>
</tr>
<tr>
<td>μ</td>
<td>viscosity, N·s/m²</td>
</tr>
<tr>
<td>θ</td>
<td>phase volume fraction</td>
</tr>
<tr>
<td>ρ</td>
<td>density, kg/m³</td>
</tr>
<tr>
<td>τ</td>
<td>dimensionless time</td>
</tr>
<tr>
<td>ψ</td>
<td>air entry potential, m</td>
</tr>
</tbody>
</table>

Subscripts

a | air

| g | gas
INTRODUCTION

A large part of the northern hemisphere is covered by snow during winter. When the snow melts, a large amount of water is generated. Part of the water released is runoff, and the other part infiltrates the underlying soil, which is frozen at the time of melt. Reliable estimates of the amounts of water infiltration and distribution are important to a number of engineering applications, such as hydrological forecast and the impact of global warming.

The theory of infiltration of water into an unfrozen, unsaturated homogeneous soil is reported by Philip in a series of papers on the subject (Philip, 1957a, 1957b, & 1957c). His results show reasonable agreement with water movement monitored in homogeneous soil under laboratory conditions. The findings of numerical, laboratory and field studies have shown that the infiltration of water into frozen ground depends on a number of factors, e.g., the temperature of the soil, the hydraulic properties of the soil, the soil moisture/ice content, and the formation of layers of basal ice (Granger, et al., 1984, Gray, et al., 1985, Flerchinger & Saxton, 1989). The entry and movement of liquid water within a frozen soil are affected by simultaneous heat and mass transfers with phase changes. These processes are coupled. The water flow velocity is directly proportional to the product of the soil permeability and the capillary pressure gradient, which in frozen soils, are strongly influenced by soil temperature.

Recently, Tao and Gray (1994) and Zhao et al. (1995) reported the results of numerical studies of coupled mass and heat transfer with phase changes in frozen soil. Zhao et al. (1995) suggest that infiltration involves primarily two flow regimes, a transient regime and a quasi-steady state regime. Once the quasi-steady state regime is reached, the energy needed to increase the soil temperature at depth is supplied by latent heat released by freezing of water in the soil above. The study investigated the effects of surface saturation, initial soil saturation and initial soil temperature on the infiltration rate and on vertical profiles of temperature, moisture (liquid water and ice) content, and phase change rate. This paper uses the formulations and numerical method described by Zhao et al. (1995) to study the effects of soil properties, initial and boundary conditions on the propagation speed and the depth of penetration of moisture and temperature waves.

Figure 1. Schematic of the frozen soil domain.

MATHEMATICAL FORMULATION

The physical system is depicted as a vertical, one-dimensional soil profile that extends from the surface, the origin, to a specified depth (see Fig. 1). The soil matrix is treated as a homogeneous porous medium. It is assumed that: (a) volumetric changes within the matrix and ice lens formation can be neglected; (b) the various soil phases are in thermal equilibrium; and (c) phase changes involving the vapor phase are small. Regarding the last, Zhao et al. (1995) reported that they are three orders of magnitude smaller than the phase changes between liquid water and ice during infiltration. The problem is formulated as one-dimensional, transient, simultaneous mass and heat flow in unsaturated soil with phase change. The non-dimensional equations for the intrinsic phase-averaged variables under the specified assumptions using the local volume averaging technique (Whitaker, 1977; Cheng 1978) are listed below.

Energy equation:

$$\frac{\partial(D_{\rho\phi})}{\partial t} + \frac{\partial \hat{\rho}}{\partial z} + P_{\tilde{m}} = \frac{\partial}{\partial z} \left( \frac{1}{P_e} \frac{\partial \hat{\rho}}{\partial z} \right).$$

(1)

Note: the definitions for the various symbols are given in the section on Nomenclature.

The liquid phase continuity equation:

$$\frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial \tilde{q}}{\partial z} - \tilde{m}_l = 0.$$  

(2)

The ice phase continuity equation:

$$\frac{\partial \tilde{\rho}}{\partial t} + \frac{1}{D_i} \tilde{m}_i = 0.$$  

(3)

The liquid phase momentum equation:
\[ \nu_I = \frac{K_1}{K} \left( \partial \rho_e \partial + 1 \right). \] (4)

The volumetric constraint:
\[ \theta_v + \theta_I + \theta_d + \theta_s = 1. \] (5)

The variables in Eqs. 1-5 are defined as:
\[ \phi = \frac{T - T_i}{T_0 - T_i}, \quad \tau = \frac{t}{t_i}, \quad \frac{V_I}{V} = \frac{V_I}{K} \rho_1 g \mu_1 L, \]
\[ \nu_I = \frac{V_I}{K} \rho_1 g \mu_1, \quad z = \frac{Z}{L}, \] (6)
\[ \dot{m}_I = \frac{M_{I}}{K \rho_1 g / L}, \quad \frac{M_{I}}{K \rho_1 g / L}, \quad P_e = \frac{P_e}{\rho_1 g L}, \]
and the parameters as:
\[ D_c = \frac{\rho c_p}{\rho I}, \quad D_I = \frac{D_i}{\rho I}, \]
\[ P_{II} = \frac{P_{II}}{c_i(T_0 - T_i)}, \quad \text{Re} = \frac{\rho_1 K_h L}{\mu I}, \]
\[ P_e = \frac{L \rho c_i K_h}{k_{eff}} = \frac{L K \rho_1^2 c_i g}{k_{eff} \mu I}, \]
where \( K_h = \rho_1 g / \mu I \) is the saturated hydraulic conductivity of the soil. The maximum liquid water content for a specific sub-zero temperature is given by the freezing point depression equation (Flerchinger and Saxton, 1989):
\[ \theta_I = \left[ \frac{h_I(T - 273.15)}{T + cR T} \right]^{1/6}. \] (8)

The air-entry potential \( \psi \) and the pore size distribution index \( b \) are determined from soil texture (Rawls, et al. 1993).

There are six equations (Eqs. 1-5, 8) and six variables: \( \phi, v_I, \theta_I, \theta_d, \theta_s, \dot{m}_I \). The set of equations are solved simultaneously using the following relations for the thermodynamic, thermophysical, and hydraulic properties of a soil.

Effective thermal conductivity:
\[ k_{eff} = \theta_v k_s + \theta_I k_I + \theta_d k_d + \theta_s k_s + \tau_k + k_D. \] (9)

The functions, \( \tau_k \) and \( k_D \), consider the effects of the tortuosity and thermal dispersion in a porous medium on thermal conductivity. The volumetric specific heat is
\[ \rho c_p = \theta_v \rho c_v + \theta_I \rho c_I + \theta_d \rho c_d + \theta_s \rho c_s. \] (10)

The effective liquid permeability, \( K_I \), and capillary pressure, \( P_o \), are (van Genuchten, 1980):

\[ K_I = KS_e^{1/2} \left[ 1 - \left( 1 - S_e^{1/m} \right)^m \right]^2, \] (11)
\[ P_c = \frac{\rho_i g}{\beta} \left[ S_e^{1/m} - 1 \right]^{1/n}, \] (12)

where \( K \) is saturated permeability and \( \beta, n, \) and \( m = 1 - 1/n \) are empirical constants. The effective saturation, \( S_e \), is defined as:
\[ S_e = \frac{\theta_I - \theta_r}{\theta_s - \theta_s - \theta_r}. \] (13)

Note, Eq. 13 treats the ice phase as part of the porous matrix.

Equations (8), (11) and (12) are empirical or semi-empirical, whereas the remaining equations are theoretical.

The following specified initial and boundary conditions were assumed. Initially, the temperature, \( \phi \), and total moisture content (liquid water plus ice), \( S_o \), within the soil matrix are constant at \( \phi = 0 \), and \( S_o \) respectively. When water is applied to the soil, the surface temperature is set to \( \phi = 1 \) (0°C), and the moisture content at the surface to \( S_o \). At \( \phi = 1 \), all water is in liquid form (Eq. 8). The values for \( \phi \) and \( S_o \) are fixed for the duration of a simulation. A list of parameters and properties and the value used in the numerical calculations is given in Table 1. The soil properties are those for a silt clay. The standard initial and boundary conditions are: \( S_o = 0.35, S_i = 0.60, \) and \( T_i = 271 \) K. Each parameter is changed to test its effect on the transfer processes while keeping the others constant.

### Table 1. The values of physical properties used in the study.

| \( T_i \) (K) | 271 | \( \beta \) (\( m^1 \)) | 0.423 |
| \( S_o \) | 0.6 | \( c \) (mol/kg) | 0 |
| \( S_i \) | 0.35 | \( n \) | 6 |
| \( L \) (m) | 0.5 | \( b \) | 4.472 |
| \( K \) (m²) | 5×10⁻¹⁴ | \( \psi \) (m) | -0.6 |
| \( \theta_s \) | 0.51 | | |

The equations are solved using an implicit, finite difference scheme. The central difference form is used for the internal nodes, and the backward or forward difference is used for the boundary nodes. A uniform grid size of 0.02 and an equal time step of 3.36×10⁻⁴ (10 min.) are used in the calculation. Any reductions in grid size and time step ensure a change of less than 5% in cumulative infiltration, \( I_{nf} \), which is:

\[ I_{nf} = \int_0^T I(t)dt. \] (14)
The solutions were deemed to have converged when two successive iterations produced agreement within $10^{-5}$ % in the dependent variables.

![Graph showing variations in the depth of penetration of moisture and temperature waves with time at: $S_t = 0.35$, $S_o = 0.6$, and $T_t = 271$ K.]

![Graph showing variations in propagation speed of moisture and temperature waves with time at: $S_t = 0.35$, $S_o = 0.6$, and $T_t = 271$ K.]

**RESULTS AND DISCUSSIONS**

The variations in the depth of penetration, $D$, and propagation velocity, $dD/dt$, of moisture and temperature waves with time, $\tau$, are shown in Figs. 2 and 3, respectively. The penetration depth is taken as the depth at which the soil moisture content had increased by 1% of the initial value and temperature had increased by 1% of the difference between the surface and initial values, respectively. Figure 2, shows that the depth of penetration of the temperature wave is always greater (roughly three times) than the depth of penetration of the moisture wave. Although a 10-fold increase in hydraulic conductivity reduced the difference in depth between the two waves, the depth of penetration of temperature was greater (about 1.5 times) than the depth of penetration of moisture. Figure 3 shows that the propagation speed of temperature decreased from 40 at the start of infiltration to about 6.6 at $\tau = 0.048$, whereas the speed of moisture propagation decreased from 6.6 to 2.5 in the same time interval.

The main reason for the difference in penetration of moisture and temperature waves is that moisture is supplied at the soil surface, whereas the heat used to increase soil temperature is supplied by the freezing of percolating meltwater within the soil. When liquid water reaches a specific depth in a frozen soil, some of the water freezes because the soil temperature is below zero. Most of the latent energy released by freezing is conducted deeper into the soil. As a result, the temperature wave advances ahead of the moisture wave. The effect of the propagation of the moisture wave on the movement of the temperature wave is small and it is noticeable only when the two waves are close together. Then, the faster moisture migrates, the faster temperature propagates. Conversely, the propagation of temperature exerts a strong effect on moisture propagation. It increases moisture propagation by raising the soil temperature, thereby the liquid water content (Eq. (8)), the effective saturation (Eq. (13)), the effective liquid permeability (Eq. (11)) and the velocity of flow (Eq. (4)). At the moisture wave front, the non-dimensional temperature increased from 0 to 0.5 ~ 0.6. This results in an increase in the effective saturation by 15%, and doubling of the relative permeability doubled due to its non-linear relationship with the effective saturation. The faster the temperature wave propagates, the higher the temperature at the moisture wave front, and the faster the moisture wave propagates.

**The Effect of Surface Saturation**

The effects on the depth of penetration and the propagation speed of moisture ($D_{Mo}$, $dD_M/dt$) and temperature ($D_T$, $dD_T/dt$) of three levels of surface saturation, $S_o = 0.5, 0.6$ and 0.7, are demonstrated in Figs. 4 and 5. Increasing surface saturation increases the propagation speed and the depth of penetration of moisture but has little effect on the propagation of temperature. Figure 4a shows that at $\tau = 0.03$, the increase in $S_o$, from 0.5 to 0.7, increased the depth of penetration of moisture from 0.16 to 0.23, an increase of 41%, and the propagation speed from 2.48 to 3.72, a change of 50%. This is because that increasing surface saturation increases the capillary pressure gradient. Conversely, as shown in Fig. 5, the penetration depth of the temperature wave increases by only 5% when $S_o$
increases from 0.5 to 0.7, and the propagation velocity remains the same at $\tau = 0.03$.

![Graph](image)

Figure 4. Variations in (a) the depth of penetration and (b) the speed of propagation of moisture waves with time for $S_0 = 0.5, 0.6$ and 0.7 at: $S_i = 0.35$ and $T_i = 271$ K.

The results in Fig. 4(b) show that the effects of $S_0$ on the speed of propagation of moisture vary. At the start of infiltration $dN_{S_0}/dt$ for $S_0 = 0.6$ is almost the same as the speed for $S_0 = 0.5$. With increasing time it approaches the velocity for $S_0 = 0.7$. This occurs because the two parameters that control the liquid velocity, namely the capillary pressure and soil permeability, are nonlinear functions of the liquid content, which is determined by the temperature. An increase in temperature causes the liquid water content to increase, which increases the permeability and decreases the capillary pressure gradient. The interaction between the propagation of temperature and the propagation of moisture is complicated. If the temperature increase is small, moisture propagation is enhanced; conversely, if the temperature increase is large, moisture propagation is impeded.

![Graph](image)

Figure 5. Variations in (a) the depth of penetration and (b) the speed of propagation of temperature waves with time for $S_0 = 0.5, 0.6$ and 0.7 at: $S_i = 0.35$ and $T_i = 271$ K.

The Effect of Initial Temperature

The effects of initial soil temperature on the propagation of moisture and temperature waves are shown in Figs. 6 and 7, respectively. The lower the initial temperature, the faster the speed of propagation of the temperature wave (see Fig. 7). This is due mainly to the higher temperature gradient and the higher thermal conductivity at the thermal wave front because of an increase in ice content (the thermal conductivity of ice is three times that of water). The simulations show at $\tau = 0.03$ that $D_T = 0.58$ for $T_i = 271$ K and $D_T = 0.88$ for $T_i = 265$ K.
Since the speed of propagation of the temperature wave increases with decreasing initial temperature, the differences among absolute temperatures at the moisture wave front for different levels of initial temperature are small. In the simulations, the absolute temperatures at the moisture wave front were 272 K and 271 K, respectively, for initial conditions of \( T_i = 271 \text{ K} \) and \( T_i = 265 \text{ K} \). The temperatures at the moisture wave front for the two cases differed by only one degree, even though the initial temperatures differed by six degrees. Since the moisture content (therefore the relative permeability) is decided by temperature, the effect of initial temperature on the propagation of moisture wave is small due to the small temperature difference at the moisture wave front regardless of the initial temperature. The difference on penetration depth of moisture is within 16% for all the different initial temperatures at \( \tau = 0.03 \).

The Effect of Initial Saturation

The propagation of moisture and temperature waves at three levels of initial moisture, \( S_i = 0.35, 0.45 \) and 0.55, are plotted in Figs. 8 and 9, respectively. Since the liquid water content of a frozen soil at a specific temperature is fixed (Eq. (8)), increasing the initial moisture content increases the ice content, which increases the effective thermal conductivity (Eq. (10)) and the effective saturation (Eq. (13)). However, the changes in thermal conductivity are small. Therefore the effects of initial moisture on the temperature wave are small over the range of moisture levels tested. The major effect of an increase in \( S_i \) on the speed of propagation of the temperature wave occurs at the beginning of infiltration (see Fig. 9). At \( \tau = 0.03 \), the depths of penetration of temperature for the various levels of \( S_i \) are within 14%. These findings suggest that the effective thermal conductivity of silty clay soil in the moisture range, \( 0.35 \leq S_i \geq 0.55 \) dominated by the
thermal conductivity of the soil particles. Soil particles, which comprise more than half of the soil mixture, have a thermal conductivity that is twice that of liquid water.

Figure 8. Variations in the depth of penetration and the speed of propagation of moisture waves with time for $T_i = 271$ K, 268 K and 265 K at $S_i = 0.35$, $S_o = 0.6$.

Increasing the level of initial saturation increases the effective saturation and permeability and decreases the capillary pressure gradient at the moisture wave front. Because the change to permeability is larger than the change in capillary pressure gradient (Zhao et al. (1995)), the net effect of an increase in initial moisture on the moisture wave is to increase the propagation speed and depth of penetration. Fig. 8 shows that the moisture penetration depth for $S_i = 0.55$ is 26% higher than that for $S_i = 0.35$ at $\tau = 0.03$.

Figure 9. Variations in (a) the depth of penetration and (b) the speed of propagation of temperature waves with time for $T_i = 271$ K, 268 K and 265 K at $S_i = 0.35$, $S_o = 0.6$.

Comparison with Field Data

Gray et al. (1984) reported the results of extensive field measurements of snowmelt infiltration into completely-frozen soils in Saskatchewan, Canada. Their measurements, which were made in a number of soils of various texture at different levels of temperature, surface saturation and saturation, show a general trend for seasonal infiltration to increase linearly with increasing depth of penetration of the moisture wave. Part of their data set is plotted in Fig. 10 and the best-fit linear regression for these data is:

$$INF = -0.006 + 0.114D_M$$

in which INF and $D_M$ are in meter. Also plotted on the figure are simulated values for INF and $D_M$ obtained with the initial and boundary conditions listed in Table 2. They also show infiltration increasing linearly with the depth of moisture penetration for a fixed set of conditions. However, the slope of the curve varies with initial and boundary conditions.
Table 2. Initial and boundary conditions for some typical runs.

<table>
<thead>
<tr>
<th>Run</th>
<th>$S_0$</th>
<th>$S_i$</th>
<th>$T_i$ (K)</th>
<th>$K_i$ ($\times 10^{-14}$, m$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run-2</td>
<td>0.6</td>
<td>0.35</td>
<td>271</td>
<td>5</td>
</tr>
<tr>
<td>Run-3</td>
<td>0.7</td>
<td>0.35</td>
<td>271</td>
<td>5</td>
</tr>
<tr>
<td>Run-7</td>
<td>0.6</td>
<td>0.55</td>
<td>271</td>
<td>5</td>
</tr>
</tbody>
</table>

Figure 10. The relationship between infiltration and moisture penetration depth.

CONCLUSIONS

This paper involves a numerical study of the propagation of moisture and temperature waves during infiltration of water into an unsaturated, frozen soil. It is found that the propagation speed of temperature is always higher than that of moisture, and the penetration depth of temperature is always deeper than that of moisture. This is due to the latent energy released by freezing of water within the soil. The propagation of temperature wave enhances the propagation of moisture by increasing the liquid water content at the moisture wave front. Conversely, movement of the moisture wave has small effect on the movement of the temperature wave unless the permeability of the soil is high, and the two waves are close together.

The effect of increasing surface saturation is to increase the moisture penetration depth and propagation speed. Conversely, the propagation of temperature wave is unaffected.

The lower the initial temperature, the faster a temperature wave propagates due to a higher temperature gradient. This reduces the effect of initial temperature on moisture penetration. At the moisture wave front, the temperature difference is much smaller than the initial temperature difference. The depth of penetration of the temperature wave increased by 50% when the initial temperature was reduced from 271 K to 265 K, whereas the moisture wave penetration depth changed by only 16%.

The effect of the initial moisture content of a frozen soil on the propagation of a temperature wave is small. Increasing initial saturation from 0.35 to 0.55, increased the depth of penetration by only 14%. The effect on the initial content on moisture wave penetration depth is larger due to the increase in permeability. Increasing initial saturation from 0.35 to 0.55 increased the depth of penetration by 26%.

It was found that the infiltration increases linearly with the depth of moisture penetration for a fixed set of conditions. The slope of the curve varies with initial and boundary conditions. This is consistent with the field observation.

Acknowledgment — The funding received from the Atmospheric Environment Service, Environment Canada, as part of the Global Energy and Water Experiment, GEWEX, is gratefully acknowledged.

REFERENCES


