Numerical analysis of simultaneous heat and mass transfer during infiltration into frozen ground

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Abstract

This paper concerns heat transfer and water infiltration into a homogeneous, unsaturated, frozen soil. A numerical analysis of the processes, which is based on the local volume averaging formulation of transport phenomena in porous media, is described. Simultaneous heat and mass transfers with phase changes are considered. The results of the simulations provide an insight into the mechanics of the infiltration process. They suggest that infiltration involves primarily two flow regimes, a transient regime and a quasi-steady-state regime. Once the quasi-steady-state regime is reached, the energy needed to increase soil temperature at depth is supplied by latent heat released from freezing of water in the upper layers of soil. The effects of surface saturation, initial soil saturation and initial soil temperature on infiltration are examined. © 1997 Elsevier Science B.V.

Keywords: Infiltration; Frozen soils; Numerical analysis; Phase changes

1. Introduction

In most cold regions of the northern and southern hemispheres, the ground freezes during winter. During infiltration of liquid water from snowmelt and rainfall in the spring, simultaneous water flow, heat transfer and phase changes occur in a frozen soil. A better understanding of the interactions among these phenomena is needed for the development of improved methods for estimating the rates and volumes of soil water recharge and runoff used for the control, conservation and management of water resources.

Numerous field studies on infiltration into frozen soil are reported in the literature (e.g. Granger et al., 1984 and Gray et al., 1985). These studies provide an insight into the process and suggest useful empirical expressions for estimating seasonal snowmelt

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infiltration. Unfortunately, they do not contribute significantly to a better understanding of the effects of various surface and soil conditions on the phenomenon. Studies of infiltration into frozen soils through laboratory-controlled experiments are restricted by the difficulties in (a) duplicating representative natural field conditions and (b) the measurement of physical quantities, e.g. the mass and rate of phase changes. Numerical studies of the phenomenon have focused on (a) the effects of freezing on the redistribution of water and soil temperature without external water flow (Jame and Norum, 1980; Harlax, 1973), and (b) the development of long-term forecasts of seasonal hydrological processes (Flerchinger and Saxton, 1989; Illangasekare and Walter, 1990; Lundin, 1990; Grant, 1992; Engelmark and Svensson, 1993). A rigorous numerical model for simulating infiltration into an unsaturated, frozen soil was reported by Tao and Gray (1994). They found that for a given water release rate from snow, the most important soil variables affecting infiltration are permeability, initial soil saturation, and the freezing-point depression characteristics of the soil. The process was less sensitive to soil temperature.

This paper concentrates on the coupling of heat and mass transfer processes and the influence of phase changes occurring within a frozen soil on water movement. It treats infiltration as a problem of one-dimensional, transient, simultaneous heat and mass flows with phase changes. The continuity and energy equations are obtained using the local volume averaging formulation (Whitaker, 1977). These equations are solved using an implicit, finite difference scheme assuming fixed boundary conditions at the soil surface. Parametric tests are performed on the simulations to examine the effects of initial soil saturation, surface saturation, and initial soil temperature on the amounts and rates of infiltration, heat transfer, and on the distribution of water within a frozen soil.

2. Nomenclature

- $c$: heat capacity, J kg$^{-1}$ K$^{-1}$
- $C$: solute concentration, mol kg$^{-1}$
- $D$: dimensionless ratio
- $d$: vapor diffusivity, m$^2$ s$^{-1}$
- $E$: coefficient of impedance factor
- $g$: gravity, m s$^{-2}$
- $h$: enthalpy change, J kg$^{-1}$
- $inf$: dimensionless cumulative infiltration
- $INF$: cumulative infiltration, m$^3$ m$^{-2}$
- $k$: thermal conductivity, W m$^{-1}$ K$^{-1}$
- $k_s$: hydraulic conductivity, m s$^{-1}$
- $K$: saturated liquid permeability, m$^2$
- $K_u$: unsaturated liquid permeability, m$^2$
- $L$: length scale determined by Eq. (10), m
- $m$: dimensionless phase change rate
- $M$: phase change rate, kg m$^{-1}$ s$^{-1}$
- $P$: dimensionless parameter
- $P_c$: pressure, Pa
- $Pe$: dimensionless capillary pressure
- $P_e$: capillary pressure, Pa
- $Pe$: Peclet number
- $q$: dimensionless cumulative conductive heat transfer
\[
\dot{q} \quad \text{dimensionless conductive heat transfer rate}
\]
\[
\mathcal{Q} \quad \text{cumulative conductive heat transfer, } J \text{ m}^{-2}
\]
\[
\dot{Q} \quad \text{conductive heat transfer rate, } J \text{ s}^{-1} \text{ m}^{-2}
\]
\[
R \quad \text{gas constant, } J \text{ kg}^{-1} \text{ K}^{-1}
\]
\[
\text{Re} \quad \text{Reynolds number}
\]
\[
S \quad \text{saturation}
\]
\[
S_c \quad \text{effective saturation}
\]
\[
Sc \quad \text{Schmidt number}
\]
\[
i \quad \text{time, s}
\]
\[
i_{\text{tot}} \quad \text{total time, h}
\]
\[
T \quad \text{temperature, K}
\]
\[
T' \quad \text{dimensionless temperature}
\]
\[
v_1 \quad \text{dimensionless liquid velocity}
\]
\[
V_l \quad \text{liquid velocity, m s}^{-1}
\]
\[
z \quad \text{dimensionless length}
\]
\[
Z \quad \text{length, m}
\]
\[
\lambda \quad \text{pore-size distribution index}
\]
\[
\mu \quad \text{viscosity, } N \text{ s m}^{-2}
\]
\[
\phi \quad \text{soil porosity}
\]
\[
\theta \quad \text{phase volume fraction}
\]
\[
\rho \quad \text{density, kg m}^{-3}
\]
\[
\tau \quad \text{dimensionless time}
\]
\[
\psi_0 \quad \text{air entry potential, m}
\]

**Subscripts**

\[
o \quad \text{surface}
\]
\[
a \quad \text{air}
\]
\[
eff \quad \text{effective}
\]
\[
g \quad \text{gas}
\]
\[
i \quad \text{ice, initial}
\]
\[
l \quad \text{liquid}
\]
\[
v \quad \text{vapor}
\]
\[
il \quad \text{from ice to liquid}
\]
\[
iv \quad \text{from ice to vapor}
\]
\[
lv \quad \text{from liquid to vapor}
\]
\[
p \quad \text{soil solid phase}
\]

### 3. Formulation

The process is formulated as one-dimensional, transient, simultaneous heat and mass flows with phase changes in a frozen, unsaturated, homogeneous soil using the local volume averaging technique (Whitaker, 1977). In developing the analysis, the following assumptions are made.

1. Within the local averaging volume, all phases (soil particle, ice, liquid water, and the gas mixture of water vapor and dry air) are in thermal equilibrium, i.e. all phases have the same temperature.
2. Convective transport in the gas phase and thermal energy transport associated with the gas phase diffusion are negligible.
3. Liquid–ice, liquid–vapor, and ice–vapor interfaces coexist within the local averaging.
volume. The length scales of the liquid–vapor and ice–vapor interfaces are proportional to the liquid and ice volume fractions respectively (Tao and Gray, 1994).

The first assumption is valid when flows and phase changes within a porous medium are moderate. That is, the temperature differences among different phases are much smaller than the global temperature scale. The second assumption is justified because the gas density is three magnitudes smaller than the density of water and the diffusion velocity is low. The third assumption imposes the constraints necessary to close the formulation. It implies that the rates of phase change between liquid and vapor and ice and vapor are proportional to the volumetric contents of liquid and ice.

The coordinate system of the soil domain is depicted in Fig. 1 in which the positive direction of the Z-axis is downward with the origin at the ground surface. The non-dimensional equations for the intrinsic phase-averaged variables under the specified assumptions are listed below.

The energy conservation equation is

$$\frac{\partial (D_e^T T_e^\ast)}{\partial \tau} + v_i \frac{\partial T_e^\ast}{\partial z} + p_l \dot{m}_i + p_i \dot{m}_s + p_v \dot{m}_s = \frac{\partial}{\partial z} \left( \frac{1}{P_e} \frac{\partial T_e^\ast}{\partial z} \right)$$

The liquid phase continuity equation is written as

$$\frac{\partial \theta_l}{\partial \tau} + \frac{\partial v_i}{\partial z} + (\dot{m}_i - \dot{m}_s) = 0$$

The ice phase continuity equation is written as

$$\frac{\partial \theta_i}{\partial \tau} + \frac{1}{D_i} (\dot{m}_i + \dot{m}_s) = 0$$

The liquid phase momentum equation is

$$v_i = \frac{K}{D_i} \left( \frac{\partial p_z}{\partial z} + (1 - D_g) \right)$$

The vapor phase diffusion equation is

$$\frac{\partial (D_e \theta_v)}{\partial \tau} - \dot{m}_i - \dot{m}_s = \frac{\partial}{\partial z} \left[ \frac{D_g}{\text{ReSc}} \frac{\partial \left( \frac{D_v}{D_g} \right)}{\partial z} \right]$$

Fig. 1. Schematic diagram of the frozen soil domain.
From assumption 3, the phase change constraint is

\[ \dot{m}_{iv} = \frac{\theta_i}{\theta_1} \dot{m}_{iv} \]  

(6)

The volumetric constraint is

\[ \theta_1 + \theta_\ell + \theta_c = \phi \]  

(7)

The variables in Eqs. (1)–(7) are defined as

\[ T^e = \frac{T - T_1}{273.15 - T_1}, \quad \tau = \frac{t K\rho_1 g}{\mu_1 L} = \frac{t}{L/k_s} \]  

(8)

\[ y_i = \frac{V_i}{K \rho_1 g / \mu_1}, \quad z = \frac{Z}{L} \]

\[ \dot{m}_{il} = \frac{M_{il}}{K \rho_1^2 g / \mu_1 L} = \frac{M_{il}}{k_s \rho_1 / L}, \quad \dot{m}_{iv} = \frac{M_{iv}}{K \rho_1^2 g / \mu_1 L} = \frac{M_{iv}}{k_s \rho_1 / L} \]

\[ \dot{m}_{lv} = \frac{M_{lv}}{K \rho_1^2 g / \mu_1 L} = \frac{M_{lv}}{k_s \rho_1 / L}, \quad p_e = \frac{P_c}{\rho_1 g L} \]

and the parameters are defined as

\[ D_{cp} = \frac{(\rho c)_e f}{\rho_1 c_1}, \quad D_g = \frac{\rho_g}{\rho_1}, \quad D_i = \frac{\rho_i}{\rho_1}, \quad D_v = \frac{\rho_v}{\rho_1} \]  

(9)

\[ Pe = \frac{L \rho_1 c_1 k_s}{k_{eff}}, \quad P_{il} = \frac{h_{il}}{c_1 (273.15 - T_1)}, \quad P_{iv} = \frac{h_{iv}}{c_1 (273.15 - T_1)} \]

\[ P_{iv} = \frac{h_{iv}}{c_1 (273.15 - T_1)} \quad \text{Re} = \frac{\rho_i k_s L}{\mu_1}, \quad \text{Sc} = \frac{\rho_i D_{eff}}{\mu_1} \]

where the length \( L \) is determined from

\[ L = 3.44 \sqrt{\frac{k_s t_{100}}{\rho c_p}} \]  

(10)

which is the temperature penetration depth (1% change in initial temperature) by heat conduction into semi-infinite solid in a total time \( t_{100} \), and

\[ k_s = K \rho_1 g / \mu_1 \]  

is the saturated hydraulic conductivity of the soil.

The maximum liquid water content at a specific sub-zero temperature is given by the freezing point depression equation:

\[ \theta_i = \phi \left[ \frac{h_{il} (T - 273.15) / T + CRT}{g \psi_0} \right]^{-\lambda} \]  

(12)

where the air-entry potential \( \psi_0 \), and the pore-size distribution index \( \lambda \) are determined from a moisture-release curve or approximated from soil physical properties using
empirical expressions. A discussion of Eq. (12) for describing the relationship between $\theta_i$ and $T$ is presented in Appendix A.

The relevant thermodynamic relations are

$$P_c = R_c \rho_c T$$

(13)

$$P_a = R_a \rho_a T$$

(14)

$$\rho_g = \rho_c + \rho_a$$

(15)

$$P_g = P_c + P_a = P_{\text{atm}} = \text{const}$$

(16)

$$P_c = P_0 \exp \left[ - \frac{P_c}{\rho_c R_c T} - \frac{h_{1c}}{R_c} \left( \frac{1}{T} - \frac{1}{273.15} \right) \right]$$

(17)

Eq. (17) is a combination of the Kelvin equation and the Clausius–Clapeyron equation. Eqs. (1)–(7) and Eqs. (12)–(17) contain 13 variables: $T^*, v_i, \theta_i, \theta_g, m_i, m_v, m_i, \rho_s, \rho_g, \rho_v, P_v$, and $P_c$. The set of equations was solved using the following empirical expressions to estimate the thermophysical and hydraulic properties of a soil.

The effective thermal conductivity and the volumetric specific heat are computed from (Whitaker, 1977)

$$k_{\text{eff}} = \theta_p k_p + \theta_i k_i + \theta_g k_g$$

(18)

and

$$(\rho c)_{\text{eff}} = \theta_p \rho_p c_p + \theta_i \rho_i c_i + \theta_g (\rho_c c_v + \rho_g c_g)$$

(19)

The effective vapor mass diffusivity is given by (Tao and Gray, 1994)

$$D_{\text{eff}} = \frac{\theta_g D}{\tau} + D_D$$

(20)

where $D_D$ is a function that accounts for the effects of mass dispersion.

Liquid permeability is calculated by the expression suggested by Brooks and Corey (1966), which is adjusted by an impedance factor to account for the presence of ice (Jame and Norum, 1980; Guymon et al., 1981). That is

$$K_l = 10^{-EB} K S_c^{3.1 + 2/\lambda}$$

(21)

in which $E$ is the coefficient of impedance factor. Values for $E$ ranging from 4.5 to 15 are reported in the literature. However, Lundin (1990) reported that he found no measurements of the value of $E$. The exponent varies with soil texture. This study used $E = 7$.

Capillary pressure is calculated by (Brooks and Corey, 1966)

$$P_c = \rho_l g \psi_0 S_c^{-1/\lambda}$$

(22)

where

$$S_c = \frac{\theta_i - \theta_r}{\phi - \theta_i - \theta_r}$$

(23)

Note, the effective saturation $S_c$ (Eq. (23)) treats the ice phase as part of the solid matrix
(see Appendix A). Saturation $S$ is

$$S = \frac{\theta_1 + \theta_i}{\phi}$$

(24)

Dimensionless cumulative infiltration $\text{inf}$ is

$$\text{inf} = \frac{\text{INF}}{L} = \frac{\int_0^L V_i(t, 0) \, d\tau}{L} = \frac{\int_0^L v_i(\tau, 0) \, d\tau}{L}$$

(25)

Dimensionless heat transfer rate at the surface $\dot{q}$ is defined as

$$\dot{q} = \frac{\dot{Q}}{k_i(273.15 - T_i)/L} = \frac{k_{\text{eff}}}{k_i(273.15 - T_i)/L} = \frac{k_{z=0}}{k_1} \frac{\partial T^*}{\partial z} \bigg|_{z=0}$$

(26)

where only conduction is considered. Dimensionless cumulative heat transfer is

$$q = \frac{\int_0^1 \dot{Q} \, d\tau}{k_i(273.15 - T_i)/k_h} = \int_0^1 \dot{q} \, d\tau$$

(27)

The equations can be solved with specified initial and boundary conditions. Initially, the temperature and total water content (liquid water plus ice) throughout the soil matrix are assumed constant at $T^* = 0$ ($T_i$ below $0^\circ C$), and $S_0$ respectively. When water is applied to the soil surface, the surface temperature is set to $T^* = 1$ ($0^\circ C$), the temperature of wet snow (Illangasekare and Walter, 1990), and the surface moisture content to $S_0$, which is higher than $S_i$. At $T^* = 1$, all water is in liquid form (Eq. (12)). The values for these parameters are fixed for the duration of the simulation. A list of values for the parameters and properties used in the numerical calculations is given in Table 1. The soil properties are those for a silty clay loam (Rawls et al., 1982).

The governing equations are solved using an implicit, finite difference scheme. The power law scheme is used to solve the energy Eq. (1) for temperature. For the remaining equations, the central difference form is used for internal nodes, and the backward or forward difference is used for boundary nodes. At every time step, the variables were solved one by one throughout the domain. The solutions were deemed to have converged when two successive iterations produced agreement within $10^{-3} \%$ in the dependent variables. Considering the balance among accuracy, economy of computing time, and

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$ ($m^2$ $cm^{-1} h^{-1}$)</td>
<td>$7.44 \times 10^{-14}$ ($0.15$)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$0.18$</td>
</tr>
<tr>
<td>$\psi_0$ ($m$)</td>
<td>$0.7033$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$0.49$</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>$0$</td>
</tr>
<tr>
<td>$S_0$</td>
<td>$0.75$</td>
</tr>
<tr>
<td>$S_i$</td>
<td>$0.4$</td>
</tr>
<tr>
<td>$T_i$ ($K$)</td>
<td>$269$</td>
</tr>
<tr>
<td>$C$ ($mol kg^{-1}$)</td>
<td>$0$</td>
</tr>
<tr>
<td>$E$</td>
<td>$7$</td>
</tr>
</tbody>
</table>
stability requirement, a uniform grid of $\Delta z = 0.02$ (\(\Delta Z = 1.36\) cm for \(L = 0.68\) m) and $\Delta \tau = 0.000368$ (10 min) is used in all of the calculations. Reducing both the time and special grid sizes changes the amounts of cumulative infiltration and heat transfer by less than 5%.

4. Results and discussions

4.1. The infiltration process

For the trial simulations, the soil is assumed to be initially at a uniform temperature of $T_i = -4$ °C, ($T^* = 0$) and a uniform level of saturation $S_i = 0.4$. The liquid content is determined by the freezing point depression expression (Eq. (12)) and the remaining water is in the form of ice. When water from snowmelt or rain is applied to the frozen soil surface, the surface temperature is set to $T_i = 0$ °C ($T^* = 1$), and the water content at the surface to $S_0 = 0.75$. Note, because the temperature is 0 °C, all water at the surface is in liquid form (Eq. (12)). The values for these parameters are fixed for the duration of the simulation, that is until $t = t_{tot} = 24$ h. From Eq. (10), the depth $L = 0.68$ m. The results of the simulations are presented in dimensional forms in the following sections.

Fig. 2 shows variations in infiltration INF and infiltration rate (the liquid velocity at the surface) $dINF/dt$ with time. Following the application of water, the infiltration rate decreases, at first rapidly and after $t = 5$ h it changes at a relatively slow, gradual rate. This pattern is similar to the entry and movement of water in an unfrozen, homogeneous soil with a constant head of water at the surface.

The variations in the conductive heat transfer rate $Q$ and total conductive heat transfer $Q$ at the surface are shown in Fig. 3. Since (a) the convection heat transfer rate at the surface is less than 5% of the conduction heat transfer rate in the transient regime and less than 1% of the conduction in the quasi-steady-state regime and (b) the total heat convection is less than 1% of the total heat conduction in the quasi-steady-state regime, the conductive heat transfer provides a good representation of the total heat transfer. The conductive heat

![Fig. 2. Variations in INF and dINF/dt with time for $S_i = 0.4$, $S_0 = 0.75$, and $T_i = 265$ K.](image)
transfer rate decreases rapidly from the start of infiltration and at $t = 5$ h reaches a relatively constant value that is about 17% of the initial value. The variations in infiltration and heat transfer rates with time suggest that the infiltration process may be described by two flow regimes: a transient period when the rates experience large decreases and a quasi-steady-state regime where the time rates of change in infiltration and heat transfer rates are relatively constant. Since the infiltration rate is directly proportional to the product of the liquid permeability and the pressure gradient, the trend can be attributed to the changes in these two variables.

Water infiltrates a frozen soil under the combined influence of capillary and gravity forces. As water enters a soil it increases the local effective saturation $S_e$, which increases the liquid permeability and decreases the capillary pressure gradient (see Eqs. (21) and
Fig. 4 plots the changes in these parameters at the soil surface with time. The relative liquid permeability increases from about 2.3 x 10^{-4} immediately following the application of water to about 1.3 x 10^{-3} at t = 5 h. Thereafter, it increases slowly with time. Conversely, the capillary pressure gradient decreases rapidly from 5.73 x 10^{3} kPa m^{-1} to 1.95 x 10^{3} kPa m^{-1} at t = 5 h, and then it decreases slowly with time. The pressure gradient due to gravity is constant at 9.8 kPa m^{-1}. In the interval 0 to 5 h, less than 1% of the infiltration rate is due to gravity. The influence of gravity increases slightly with time. At t = 24 h, it is 2%.

The results of an energy budget for a control volume at Z = 0.04 m are shown in Fig. 5. The temperature, the rate of change in internal energy, and the energy transfer rates by conduction, by phase-change and by convection are plotted with time in Fig. 5(a) and the changes in total energy with time in Fig. 5(b). In these figures a positive value indicates that energy is added to the system. During the early stages of infiltration, energy is added to the control volume mainly by conduction. This energy is used to increase the internal energy (temperature) and to melt ice (a negative value of energy change due to phase change). At t = 5 h, when the wetting front reaches the layer, the conductive heat transfer rate and the energy change rate due to the phase-change are zero and the local temperature has increased from -4 °C to -1.34 °C (Fig. 5(a)). In the increment, t = 0 to t = 5 h, about 62% of the conducted energy is used to melt ice and the remainder to raise the local temperature. When t > 5 h, heat is added to the control volume by latent energy released by freezing of liquid water. More than 90% of this latent heat is conducted deeper into the soil. The cumulative energy contributed to the control volume by the phase-change is a minimum at t = 5 h and zero at t = 8.5 h, whereas the net heat transferred into the control volume by conduction is a maximum at t = 5 h and zero at t = 10.5 h (see Fig. 5(b)). At t = 10.5 h, the total energy used to increase the local temperature is equal to the sum of the energy released from freezing plus the energy supplied by convection. At t > 10.5 h, the net heat input by conduction to the control volume is negative. This means that after a certain time, all the energy needed to increase the local temperature from the initial value to its existing value is supplied by the latent heat released from freezing of liquid water and convective heat transfer, and the extra latent heat released from freezing is conducted downward into the soil below. Once the quasi-steady-state is reached, the energy needed to increase the soil temperature at depth is not supplied from the soil surface, but from the latent heat released from the freezing of liquid water in the soil. The data in Fig. 5(a) and (b) also show that the convective heat transfer rate is usually one magnitude smaller than the conductive and phase change rates. However, the cumulative contribution to the internal energy rise by convection cannot be neglected because it increases with time and at t = 24 h is about 27%.

Variations in the vertical profiles of temperature T (°C), rate of phase change between ice and liquid water \( M_d \) (kg h^{-1} m^{-3}), ice volume content \( \theta_i \), and saturation \( S \), with time are plotted in Fig. 6. As described above, heat is transferred into a frozen soil mainly by conduction. When the temperature wave reaches a given depth, part of this energy is used to raise the local temperature of the soil and part is used to melt the ice. Melting occurs because the total moisture (liquid water plus ice) content is constant, and the amount of liquid water produced by the increase in temperature is determined by the temperature depression relationship (Eq. (12)).
Fig. 5. Variations in (a) energy change rate and temperature, and (b) total energy with time at $Z = 0.04$ m with $S_e = 0.4, S_0 = 0.75$, and $T_i = 269$ K.

As shown in Fig. 6(a) and (b), the temperature wave is always ahead of the wetting front. The heat transfer process is driven by latent heat that is released owing to freezing of percolating meltwater in the soil above (because the temperature of the soil is less than 0°C). Also, the increase in local temperature is augmented by the latent energy released by freezing of water within the soil layer (negative $M_d$ in Fig. 6(b)) when the percolating water reaches the layer. Figs. 6(b) and (c) show a melting wave (positive $M_d$ in Fig. 6(b)) at the thermal wave front (defined as temperature changed by 1% of $T_0 - T_i$, that is 0.04°C), a freezing wave (negative $M_d$ in Fig. 6(b)) in the soil layer immediately above the wetting front, and another melting wave nearer the surface. The last is supplied by conductive
energy from the soil surface. Accordingly, Fig. 6(c) shows that the ice content has a minimum due to melting and a maximum due to freezing. At $t = 24 \text{ h}$, the temperature wave had penetrated to 0.45 m. The saturation curve (Fig. 6(d)) shows that the depth at which soil moisture increased by 1% was much less, about 0.12 m. The decreases in the rates of vertical elongation in the temperature wave and the wet front (Fig. 6(d)) with
increasing time indicate that the speeds of propagation of the fronts decrease with time. Once quasi-steady-state conditions have been reached, the degree of saturation of the soil layer adjacent to the surface remains relatively constant. The levels are higher than at the surface because of the higher ice content (Fig. 6(c)). However, the liquid content is always the highest at the surface.

The simulations also showed that during infiltration into frozen soil, the phase change rates between ice and vapor and between liquid and vapor are at least three magnitudes (usually four to five magnitudes) smaller than the phase change rate between ice and liquid. In general, phase changes involving vapor have a negligible effect on the infiltration process in frozen soils.

4.2. The effect of surface saturation $S_0$, initial saturation $S_i$, and initial temperature $T_i$

4.2.1. The effect of surface saturation $S_0$

With other parameters held constant, as the surface saturation (defined by Eq. (24) and used as boundary condition) decreases, the capillary pressure gradient and the permeability of the soil at the surface decrease. These decreases lower the infiltration rate and the amount of infiltration, as well as the depth of penetration of the wetting front. In contrast, the total heat transfer at the surface increases with decreasing $S_0$ (see Fig. 7). This point is discussed below.

The effects of surface saturation on profiles of soil temperature $T$, and rate of phase change between ice and liquid water $M_{iir}$, at $t = 12$ h are plotted in Fig. 8. The curves in Fig. 8(a) show that the depth of penetration of the temperature wave increases with increasing $S_0$. Also, the amplitude of the freezing wave becomes stronger (Fig. 8(b)). These results suggest that the portion of the total energy required for the increase in soil temperature supplied by the phase change, relative to the amount supplied by conductive heat transfer from the soil surface, increases with increasing $S_0$. As more energy is supplied by latent heat, less energy is transferred into the soil by conduction (Fig. 7).

![Graph showing variation of Q with time for different surface saturations](image)

Fig. 7. Variation in conductive heat transfer at the soil surface $Q$ with time for $S_0 = 0.60, 0.75$ and $0.90$ at $S_i = 0.4$ and $T_i = 269$ K.
Fig. 8. Profiles of (a) temperature $T$, and (b) rate of phase change $\dot{M}_p$, for $S_0 = 0.60, 0.75$ and 0.90 at $t = 12$ h and $S_i = 0.4$, $T_i = 269$ K.

Fig. 9. Profiles of (a) change in soil saturation $\Delta S$, and (b) rate of phase change $\dot{M}_p$, for $S_i = 0.4, 0.5$ and 0.6 at $t = 12$ h and $S_0 = 0.75$, $T_i = 269$ K.
4.2.2. The effect of initial saturation $S_i$

An inverse relationship between infiltration and moisture/ice content in frozen soils has been postulated by many investigators (e.g. Willis et al., 1961; Kuzik and Bezmenov, 1963; Motovilov, 1979; Granger et al., 1984). Our simulation data substantiate these findings. As the initial saturation increases, the capillary pressure gradient decreases, which causes infiltration to decrease. Fig. 9(a) plots the changes in saturation level at various depths at $t = 12$ h. The data show that most of the infiltrated water is concentrated in the soil layers near the surface and the depth of penetration of the infiltration front is relatively independent of initial saturation $S_i$. However, more water infiltrates the drier soil. This pattern is consistent with the findings of Granger et al. (1984) and Gray et al. (1986) who observed that the average depth that infiltrating meltwater penetrated completely frozen, uncracked soils of various textures (sands, silts and clays) during snowmelt ablation in the Canadian Prairies was of the order of 0.26 m (standard deviation 0.1 m).

Fig. 9(b) shows variations in the rate of phase change between ice and water with depth for three levels of $S_i$. The drier the soil, the greater the amplitude of the freezing wave, i.e. the greater the phase change rate $M_{ij}$; therefore, the smaller the amount of energy conducted into the soil at the surface.

4.2.3. The effect of initial temperature $T_i$

Komarov and Makarova (1973), Steenhuis et al. (1977), and Granger et al. (1984) suggest that the effects of soil temperature at melting on snowmelt infiltration may be secondary to other parameters, such as the soil moisture content. Fig. 10 plots the variation in cumulative infiltration with time for three levels of initial temperature and a constant moisture content. The data show that the higher the initial temperature, the larger the cumulative infiltration. The decrease in infiltration with decreasing initial temperature is caused by the increase in impedance and consequent decrease in permeability (Eq. (21)) due to increasing ice. For an extreme case with $E = 0$, INF at $t = 24$ h for the three cases $T_i = -4, -6, \text{ and } -8 ^\circ$C, agreed within 6%.

Although important, the effects of initial temperature on infiltration are relatively small compared with those produced by changes to other variables, e.g. initial saturation and surface saturation. At $t = 24$ h, a reduction in initial temperature from $-4$ to $-8 ^\circ$C

![Fig. 10. The variations in INF with time for $T_i = -4 ^\circ$C, $-6 ^\circ$C and $-8 ^\circ$C at $S_i = 0.4$, $S_o = 0.75$.](image-url)
decreased infiltration by 19%. In comparison, infiltration decreases by 52% when the initial saturation is increased from $S_i = 0.4$ to $S_i = 0.6$ and increases by 143% when the surface saturation is increased from $S_0 = 0.75$ to $S_0 = 0.90$.

Profiles of the temperature and changes in ice content at $t = 24$ h for three values of $T_i$ are plotted in Fig. 11. They show that the lower the initial temperature the larger the change in ice content. Because of the larger amount of latent heat supplied by the freezing of water and a larger temperature gradient, the temperature of a cold soil increases faster. After 24 h the soil temperature at the wetting front had increased to $-1$ °C for $T_i = -4$ °C, $-1.2$ °C for $T_i = -6$ °C, and $-1.7$ °C for $T_i = -8$ °C.

5. Conclusions

This paper provides a numerical analysis of simultaneous heat transfer, liquid water flow, and phase change in unsaturated, frozen soil during infiltration. The governing equations were based on a local volume averaging formulation of transport phenomena in porous media. Solutions were obtained iteratively using an implicit, finite difference method.

The results of the simulations suggest that the infiltration process may be described by two regimes, a transient period and a quasi-steady-state period. The transient period follows immediately the application of water and during this period the infiltration rate and heat transfer rate decrease rapidly. Also, during the transient period the liquid permeability experiences large increases, and the capillary pressure gradient decreases rapidly.

Fig. 11. Profiles of (a) temperature $T$, and (b) change in ice content $\Delta\theta_i$, for $T_i = -4$ °C, $-6$ °C, and $-8$ °C at $t = 24$ h and $S_i = 0.4$, $S_0 = 0.75$. 
The transient regime is followed by a quasi-steady-state period in which the infiltration rate changes gradually with time at a slow rate, and the surface heat transfer rate approaches a relatively constant value.

Initially, the temperature of a layer of frozen soil is increased by energy transported by the conduction of heat from the soil that overlies the layer. This process is associated with the melting of ice. As percolating water reaches the layer, the increase in soil temperature is augmented by the latent heat released from the freezing of liquid water. With increasing time, most of the latent energy is conducted deeper into the soil and the net receipt of energy by the layer becomes zero. The energy needed to raise the temperature is equal to the latent heat released from freezing plus the convective heat transfer at that time. This process is characterized by a melting wave at the temperature wave front, followed by a freezing wave.

The effects of phase changes involving the vapor phase can be neglected, since the rates are at least three magnitudes smaller. The effect of convective heat transfer, however, cannot be neglected. It may contribute up to 27% of the energy needed to raise the soil temperature.

The effect of increasing the surface saturation during infiltration is to increase the cumulative infiltration and to decrease the surface heat transfer. However, the effect of increasing the initial soil saturation is to reduce the cumulative infiltration and to increase the surface heat transfer. The effects of initial soil temperature on cumulative infiltration are secondary because the temperature at the front of percolating water is relatively independent of the initial temperature.

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Appendix A.

Obtaining accurate and reliable estimates of hydrodynamic properties, e.g. capillary pressure and permeability, of unsaturated soils, is difficult owing to the intricacy of soil structure. These evaluations are even more complex for a soil in the frozen state. In an unsaturated frozen soil, liquid water, ice and air co-exist in the soil pores. A description of possible configurations of the three phases in a soil is given by Miller (1980). Ice, which forms near the center of the pores, is surrounded by liquid or a mobile liquid film. According to the configurations, \( P_{\text{ice}} \geq P_{\text{air}} > P_{\text{water}} \). The ice pressure \( P_{\text{ice}} \) may or may not be equal to the pressure of air \( P_{\text{air}} \), depending on whether the ice is under stress. Assuming the air pressure (gage pressure) and the total water content (liquid plus ice) remain constant, the liquid water pressure and the liquid water content in a frozen soil increase progressively as the temperature increases. This creates a pressure difference, or 'capillary
sink', which drives fluid flow from warmer to colder regions. Since the ice is surrounded by capillary liquid or a thin mobile liquid film and the liquid is in contact with air, which is at atmospheric pressure, it is assumed that the ice plays a passive role during infiltration into frozen soil. The major effect of ice is to reduce the pore size. Assuming \( P_{\text{ice}} = P_{\text{air}} \), which is valid for the case of soil thawing with external water infiltration, permeability and the capillary pressure can be calculated by correlations such as Eq. (21) and Eq. (22), provided that an impedance factor is used to reduce the permeability due to the presence of ice.

The paper assumes an effective saturation of a frozen soil defined by Eq. (23), which includes the ice fraction. There were two reasons for adopting this definition.

1. The assumption that the major effect of ice on flow is due to its effect on pore size, hence permeability.
2. The total water content of a frozen soil is taken as the sum of the liquid and ice fractions. Therefore, when saturated, \( \theta_1 + \theta_i = \phi \), and \( S_c = 1 \). If \( S_c \) is defined in the 'normal' manner, i.e. \( S_c = (\theta_1 - \theta_i)/(\phi - \theta_i) \), only the liquid water content is considered. Therefore, for a frozen soil, \( S_c \neq 1 \) at saturation. This leads to the unrealistic situation that a capillary pressure exists and water can flow to fill space.

Eq. (12) was used to calculate the unfrozen liquid water content at a sub-zero temperature. This expression was developed by Cary and Mayland (1972) by applying the Clausius–Clapeyron equation to an air-free, liquid water–ice system in frozen soil. It is based on the assumption suggested by Miller (1965) that the soil matric potential for an unsaturated, unfrozen soil and for frozen soil are similar functions. In an unsaturated frozen soil three phases, liquid water, ice and air, co-exist. Generally speaking, the Clapeyron equation for three phases includes the pressure differences between liquid water and air and between ice and air. During infiltration into frozen soil, it is assumed that the pressure difference between ice and air is much smaller than that between liquid water and air. Under this assumption, Eq. (12) is valid as the freezing point depression equation of a three-phase system.

Theoretically, Eq. (12) should be consistent with the pressure Eq. (22) using the effective saturation defined in Eq. (23). The freezing point depression equation would involve the ice fraction, therefore the liquid water fraction–freezing temperature relationship would be a function of total water content. This contravenes the experimental findings by Jame (1972) and other researchers that the unfrozen liquid volume content is only a function of temperature. A limited number of simulations were conducted using effective saturation defined in Eq. (23) with the freezing point depression equation (Eq. (12)). The difference in cumulative infiltration is very small (less than 5%). In the present simulation, Eq. (12) is used as an empirical equation. Eq. (12) has been used by many researchers in simulating water flow in frozen soils (e.g. Fuchs et al., 1978; Flerchinger and Saxton, 1989; Tao and Gray, 1994).

References


