Evaluation of the heat-pulse probe method for measuring frozen soil moisture content

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What is Heat Pulse Probe (HPP)

\[ \Delta T = f(C, q, t_0, t, r) \]

Determines \( C \)

\[ C = \sum (C_m \theta_m + C_o \theta_o + C_l \theta_l + C_i \theta_i) \]

Determines \( \theta_l, \theta_i \)
Why HPP for soil ice?

- No other options for $\theta_i$ other than radioactive methods.
- Successfully tested in unfrozen soil.
- $\theta_i$ could be determined by TDR or $\theta_i$-$T$ curves.

HPP has the potential for soil ice measurements with:

- Relatively low cost
- Continuous measurements
- Minimum disturbance to natural conditions
How to make HPP work for frozen soil?

Problems for frozen soils:

Possible ice melting will null the assumptions of current mathematical solutions for HPP:

- All energy is used to raise soil temperature by conduction
- Soil thermal properties ($C, \lambda, \kappa$) are constant and homogenous

Possible solutions:

- Control $q, t_0$ to limit melting
- Revise the mathematical solution to include soil thawing
Lab experiments

Material:
fine sand with porosity of 0.35

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<tr>
<td><strong>Controls</strong></td>
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<td><strong>q</strong></td>
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<td>T</td>
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Mathematical solutions

1. Instantaneous Infinite Line Source (IILS)

\[
\Delta T(r,t) = \frac{q}{4\pi\lambda t} \exp\left(\frac{-r^2}{4\kappa t}\right)
\]

2. Pulsed Infinite Line Source (PILS)

\[
\Delta T(r,t) = \begin{cases}
-q' \frac{Ei\left(\frac{-r^2}{4\kappa t}\right)}{4\pi\lambda} & 0 < t \leq t_0 \\
\frac{q'}{4\pi\lambda} \left\{ Ei\left[\frac{-r^2}{4\kappa(t-t_0)}\right] - Ei\left(\frac{-r^2}{4\kappa t}\right) \right\} & t > t_0
\end{cases}
\]

3. Finite Difference Numerical Model (FDNM)

\[
C_p \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(rK \frac{\partial T}{\partial r}\right) + q' \quad C_p = C_v + \rho_L \frac{d\theta_u}{dT}
\]
The Numerical Scheme

Vertical system

Radial system
Performance of three methods in different initial temperature

\[ q \approx 900 \text{ J m}^{-1} \]

\[ t_0 = 8 \text{ seconds} \]

\[ s = 1.0 \text{ (Saturated)} \]
Performance of three methods under different heating time

\[ q \approx 1000 \text{ J m}^{-1} \]
\[ T_{\text{ini}} \approx -2.0 \, ^\circ\text{C} \]
\[ s = 1.0 \text{ (saturated)} \]
Performance of three methods under different moisture and temperature combinations

\[ q \approx 450 \text{ J m}^{-1} \]

\[ t_0 = 30 \text{ seconds} \]
Sensitivity of \( \Delta T \) to heat application

\[
q = 100 \sim 1000 \text{ J m}^{-1}
\]

\[
T_{\text{ini}} = -2 \sim -10 \degree \text{C}
\]

\[
s = 0.5
\]
Sensitivity of $\Delta T$ to total moisture content ($\theta$) under different heat applications (HPP applicability)
Error distribution of frozen moisture measurement of HPP with calibrated probe spacing (5.0-6.2mm)
Conclusions

Only the numerical model could represent the measured $\Delta T$ curves once ice melting occurs during HPP measurements.

Below -4°C, ice melting could be controlled to a limited level such that it has little effects to HPP measurements.

The measurement errors of $\theta$ were well within ±0.05 m$^3$ m$^{-3}$ under -4°C, but become unpredictable between -2°C and 0°C.

The failure of HPP between -2°C and 0°C are mainly due to the retarded response of $\Delta T$ to changing $\theta$ and $q$.

The probe spacing is a very sensitive parameter and needs recalibration each time the probe inserted into soil or the soil goes through a thawing/freezing processes.